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RADIO ENGINEERING

(RADIOTEKHNIKA)

BY

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ADVANCES AND PROBLEMS OF RADIO ENGINEERING IN THE USSR

Every 7 May the Soviet nation marks the **glorious** anniversary of the invention of radio by the great compatriot A. S. Popov.

Like other branches of science, radio engineering attained real development in the USSR only after the Great October Socialist Revolution. The attention paid by the Communist Party and by the government to the development of science and engineering, to the development of the national economy, to the improvement of the well-being of the Soviet people, to the maximum fulfillment of their material and spiritual needs -- all created the necessary foundations for a tremendous expansion of radio engineering in the USSR. During the years of the Soviet regime tremendous progress has been made in the radio industry and in the network of radio engineering research institutes, laboratories, and institutions of learning.

The historic resolutions of the Nineteenth Congress of the Communist Party of the Soviet Union and of the September Plenum of the TSK KPSS, and the decisions of the TSK KPSS and of the Soviet government concerning the development of television and UHF broadcasting, increasing the output of television and broadcast receivers for the population, and installing radio equipment in rural localities, have uncovered before Soviet radio engineering extensive prospects of further development..

The development of radio engineering is to follow three basic paths: systematic and unceasing improvement of all qualitative indices of apparatus, mastery of new bands of electromagnetic waves, and expansion of fields of application of radio methods and electronics. Considerable advances have already been made along these lines. These aims also characterize the trends of further development and determine the immediate practical tasks of Soviet radio engineering.

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Improvement in the radio-telegraph, radio-broadcasting, and television transmission systems calls for new receiving and transmitting apparatus.

Further increase in the effective range, reliability, simplicity, and ease of operation of equipment requires an unceasing improvement in theory, the creation of new and original developments, search for new materials, a better manufacturing technology, and improvement in operating methods.

Modern professional receivers afford accurate reception of radio signals at received radiation intensities of 10^{-13} to 10^{-14} watts per square meter. This corresponds approximately to the radiation intensity obtained at a distance 200 to 600 kilometers from the bulb of a pocket flashlight.

The high sensitivity of radar receivers has permitted the reception of weak radar signals reflected from the moon. Greater still is the sensitivity of modern radio telescopes (i.e., devices intended for reception of radio waves from cosmic bodies) consisting of highly-directional antennas, receivers, and recording devices.

As the number of radio stations has grown and as transition has been made to ever shorter wavelengths, one of the principal processes of radio transmission, the generation of electromagnetic oscillations, has changed fourfold during the half century -- from spark transmitters of damped oscillations through arc oscillators and high-frequency machines to modern vacuum-tube oscillators.

The application of radio communication in various branches of the national economy and the development of radar and radio navigation have imposed high demands on the modern vacuum-tube oscillator and radio transmitter as a whole.

As early as 1922 Soviet specialists created the mightiest radio-broadcasting station in the world. The capacity of Soviet radio stations is still

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continuously increasing.

The first period in the development of radio engineering, from 1895 to approximately 1918, was characterized by the use of longer and longer waves. The successes of the first experiments of radio amateurs, who could communicate by radio over very large distances with transmitters having outputs of a few watts, led to intensive study of the short-wave band during the twenties.

The use of the short-wave band produced an economic and reliable solution to the problem of communication over a long distance, and also permitted the use of many channels without mutual interference. However, the continuous expansion of new communication services soon exhausted even the potentialities of this band. This circumstance, in particular, produced a great interest in the study of even shorter wavelengths. These works were spurred on by the fact that new radio services, such as television and radar, called for channels of very great bandwidths. This led in recent years, and particularly during the Great Fatherland War, to the use of meter, decimeter, and centimeter waves.

There is perhaps no single branch of science or engineering that offers more examples of the linking of one branch of science with another than does radio engineering. This is explained chiefly by the flexibility of radio methods, the ease with which physical, chemical, and mechanical quantities are transformed into electrical quantities, the universality of electric instruments and their freedom from inertia, and the possibility of remote control and automatization of the apparatus.

An idea of the volume of work and tasks of the radio industry can be had from the resolutions adopted in October 1953 by the Council of Ministers of the USSR and by the Central Committee of the KPSS concerning increased production and improvement of the quality of industrial products for mass consumption.

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Thus, 2,861,000 radio receivers and 325,000 television sets are to be produced in 1954, and 3,767,000 radio sets and 760,000 television sets are to be produced in 1955. At the same time the number of models is to be increased, the quality and external appearance improved, and the cost lowered.

An essential condition for a sharp increase in production and for the resultant decrease in cost is full utilization of the internal reserves of the plants -- mechanization and automatization of manufacture, and employment of progressive technology. For example, an analysis of the cost of radio sets for mass consumption shows that it is determined principally by the cost of parts and radio tubes. It follows therefore that the manufacture of radio parts, of attachments, and of individual units should be organized in individual specialty enterprises. Production in these enterprises should be automatized and mechanized to the limit; the designs of the parts and units should be normalized and standardized.

The continuous growth of the economic and engineering might of our Fatherland, and the mighty accomplishments of Soviet science create exceptional potentials for further creative workby Soviet scientists and engineers in the building of a communist society.

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OPTIMUM TRANSITION BETWEEN TWO DIFFERENT UNIFORM LONG LINES

by

P. I. Kuznetsov and R. L. Stratonovich

This work gives a solution to the problem of selecting the optimum matching section between two uniform lines, using a section of a non-uniform line.

This work contains a solution to the problem of finding the best equation for the variation of the parameters of a non-uniform line that matches two uniform lines with characteristic impedances ρ_1 and ρ_2. To get the minimum reflection in the transition piece, it is natural to make the non-uniform line as long as possible. However, this length is usually limited by technical consideration. Let ℓ be the maximum permissible length. We shall seek the best transition function for prescribed values of ℓ, ρ_1, and ρ_2. For symbols and numbering of equations we refer the reader to our earlier work (1) .

1. Before proceding to solve the problem posed above, let us consider several questions associated with the reflection coefficient:

$$P(x) = \frac{Z_x - P(x)}{Z_x + P(x)}, \qquad (1)$$

which satisfies the Riccati equation

$$\frac{d\rho}{dx} + 2\tau\rho + N\rho^2 - N = 0. \qquad (2)$$

The boundary contition for this equation is obtained by setting $X = 0$ in (1)

$$P(0) = \rho_0 = \frac{Z_H - \rho(0)}{Z_H + \rho(0)}. \qquad (3)$$

The boundary value p_0 is the reflection coefficient directly at the load, due to the fact that the load does not match the termination of the line.

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Let us assume first that the line is loaded at the point $x = 0$ with an impedance equal to the characteristic impedance $\rho(0)$. Then there will be no reflection from the load. Let us denote the reflection coefficient from the non-uniform line for that case, by p_1 ($p_1 = U_{ref} / U_{inc}$), and the transmission coefficient by q_1 ($q_1 = U_{trans}/ U_{inc}$). Now, let the generator be placed at the other side of the non-uniform line. For a wave travelling in the opposite direction, the reflection and transmission coefficients will be p_2 and $q_2 = q_1 \rho_1 / \rho_2$ respectively.

If $Z_L \neq \rho(0)$, the reflected wave in the uniform line, at $x > \mathcal{C}$, is obtained by adding the waves that are multiply reflected from the load and from the non-uniform line. Consequently, the reflection coefficient at $x > \mathcal{C}$

equals
$$p = p_1 + q_1 p_0 q_2 + q_1 p_0 p_2 p_0 q_2 + q_1 p_0 (p_2 p_0)^2 q_2 + \cdots =$$
$$= \frac{p_1 + (q_1 q_2 - p_1 p_2) p_0}{1 - p_2 p_0}. \tag{4}$$

Using the functions introduced in reference [3], the reflection coefficient can be represented in the following form

$$p(x) = \frac{a(x) + b(x) p_0}{c(x) + d(x) p_0} \tag{5}$$

with the following boundary conditions:

$$a(0) = 0; \quad b(0) = 1; \quad c(0) = 1; \quad d(0) = 0. \tag{6}$$

It is important to know these functions for $x = \mathcal{C}$, since

$$p_1 = \frac{a(l)}{c(l)}, \quad p_2 = -\frac{d(l)}{c(l)}; \quad q_1 q_2 - p_1 p_2 = \frac{b(l)}{c(l)}, \tag{6a}$$

which is a consequence of comparing (4) with (5).

Using the same method is in an earlier work[1] it is possible to prove that $p(x)$ can always be represented in the form (5) with boundary conditions (6), and to derive differential equations for $a(x)$, $b(x)$, $c(x)$, and $d(x)$. The expressions $a(x)/c(x)$ and $b(x)/d(x)$ are particular solutions of the Riccati equation (2) and consequently

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$$c(a' + 2\gamma a - Nc) - a(c' - N'a) = 0, \tag{7}$$

$$d(b' + 2\gamma b - N'd) - b(d' - Nb) = 0. \tag{8}$$

Furthermore, as in paragraph 4 of reference [1], we obtain

$$d'c - c'd = N(bc - ad). \tag{9}$$

If we require that functions $a(x)$, $b(x)$, $c(x)$, and $d(x)$ be related by the following equation:

$$c'(x) - N(x)a(x) = 0, \tag{9a}$$

we obtain, by taking (7), (8), and (9) into account, the following systems of equations:

$$a' + 2\gamma a = Nc; \quad c' = Na; \tag{10}$$

$$b' + 2\gamma b = Nd; \quad d' = Nb \tag{11}$$

with boundary conditions (6)

Multiplying the first equations with an integrating factor $\exp(2\gamma x)$ and integrating them, we obtain the systems of integral equations first derived using another method by A. L. Fel'dshteyn in reference [3]: these equations were solved by him using the method of successive approximations.

2. Let us find the optimum transition function between two different uniform waveguides with prescribed characteristic impedances ρ_1 and ρ_2 for a definite length $\mathcal{Q}$ of the transition link. The propagation constants of the two lines are assumed the same. We shall indicate in a subsequent work how to get rid of this limitation.

It is important in practice to have the minimum power reflected at frequencies lying within the operating band $\omega_1 < \omega < \omega_2$. The optimum transition function $N_{opt}(x)$ for the prescribed frequency band will depend on ω_1 and ω_2. We require that the integral

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$$\int W_{omp}\, d\omega \tag{11a}$$

have a minimum value when the power of the incident wave is constant for the prescribed pass band

$$W_{nad} = \frac{U_{nad}\, U_{nad}^{\bullet}}{2}\, \mathrm{Re}\, \rho_1; \tag{11b}$$

The power of the reflected wave equals

$$W_{omp} = \frac{U_{omp}\, U_{omp}^{\bullet}}{2}\, \mathrm{Re}\, \rho_1. \tag{11c}$$

In view of the fact that w_{inc} is constant, it is possible to seek a minimum of the integral

$$J = \int_{\omega_1}^{\omega_2} \frac{w_{omp}}{w_{nad}}\, d\omega = \int_{\omega_1}^{\omega_2} \frac{U_{omp}\, U_{omp}^{\bullet}}{U_{nad}\, U_{nad}^{\bullet}}\, d\omega = \int_{\omega_1}^{\omega_2} pp^{\bullet}\, d\omega. \tag{12}$$

For the reflection coefficient we shall use the first-approximation formula

$$p_1 = -e^{-2\gamma x} \int_0^l e^{2\gamma \xi} N(\xi)\, d\xi, \tag{12a}$$

cited as number (26) in reference[1].

For the normally encountered lines, $\gamma = i\omega/v$, where v is the phase velocity of the wave. Analogous computations can be carried out for the second dispersion law. Using $\gamma = i\omega/v$, we shall rewrite (26) as follows:

$$p = -e^{-2i\frac{\omega}{v}x} \int_0^l e^{2i\frac{\omega}{v}\xi} N(\xi)\, d\xi. \tag{12b}$$

Substituting this expression into (12), we get:
$$J = \int_{\omega_1}^{\omega_2} \int_0^l \int_0^l e^{2i\frac{\omega}{v}(\xi-\xi')} N(\xi)\, N^{\bullet}(\xi')\, d\omega\, d\xi\, d\xi'$$

Integrating with respect to ω and introducing new variables

$y = x-a = \xi -a$, $y' = c' - a$, $a = \ell/2$, we find

$$J = \int_{-a}^{a} \int_{-a}^{a} Q(y-y') N_1(y)\, N_1^{\bullet}(y')\, dy\, dy', \tag{12d}$$

Where

$$Q(y-y') = \frac{v}{2i\,(x-x')}\left[e^{i\frac{2\omega_2}{v}(x-x')} - e^{i\frac{2\omega_1}{v}(x-x')} \right], \quad N_1(y) = N(y+a) = N(x). \tag{12e}$$

. From now on we shall drop the index 1. The minimum of J must be found under the condition

$$\int_{-a}^{a} N(x)\, dx = \frac{1}{2}\ln\frac{\rho_1}{\rho_2}. \tag{13}$$

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$$\tag{13}$$

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Let us first restrict ourselves to the case when $\dot{r}_1 // \dot{r}_1$ is a real quantity. Then the unknown N(x) is also a real function.

Since $\dot{J}(x)$ is real, then

$$J = \int_{-a}^{a} \int_{-a}^{a} K(y - y') N(y) N(y') dy dy', \qquad (14)$$

where

$$K(y - y') = \operatorname{Re} Q(y - y') = -\frac{v}{2(y - y')}\left(\sin 2\omega_2 \frac{y - y'}{v} - \sin 2\omega_2 \frac{y - y'}{v} \right). \qquad (14a)$$

We shall use a direct method to find the minimum of J. We shall seek the optimum transition function $N_{opt}(y)$ in the form of a series

$$N(y) = \sum_{i=0}^{\infty} a_i \varphi_i(y) \qquad (15)$$

of the following orthonormal functions:

$$\varphi_0 = \frac{1}{\sqrt{2a}} \quad ; \quad \varphi_{2m} = \frac{1}{\sqrt{a}} \cos \frac{\pi m y}{a} \; , \quad \varphi_{2m-1} = \frac{1}{\sqrt{a}} \sin \frac{\pi m y}{a} \quad (m \quad 1,2,3,...) \qquad (16)$$

Substituting (15) into (14) we get

$$J = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_m a_n \int_{-a}^{a} \int_{-a}^{a} K(y - y') \varphi_m(y) \varphi_n(y') dy dy'. \qquad (17)$$

The kernel K(y-y') is continuous and bounded, and therefore it can be represented, in the domain $-a \leqslant y \leqslant a$, $-a \leqslant y' \leqslant a$, in the form of a series

$$K(y - y') = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{mn} \varphi_m(y) \varphi_n(y'), \qquad (18)$$

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where

$$c_{mn} = \int_{-a}^{a} \int_{-a}^{a} K(y-y') \varphi_m(y) \varphi_n(y') \, dy \, \omega' . \qquad (19)$$

Here $c_{mn} = c_{nm}$ since the kernel K is even.

By comparing (19) and (17) we get

$$J = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{mn} a_m a_n. \qquad (20)$$

The above operations are valid if both $N(x)$ and $K(y-y')$ can be represented

in the form of the convergent series (15).

Condition (13) assumes the form

$$a_0 = \frac{1}{2\sqrt{2a}} \ln \frac{\rho_1}{\rho_2} . \qquad (21)$$

Thus the first coefficient in the expansion (15) has been determined.

Let us find the remaining coefficients. We seek the best approximation to the

extremum among those functions that have k first harmonics:

$$N_k(y) = \sum_{n=0}^{k} a_n \varphi_n(y). \qquad (22)$$

By letting k approach infinity, i.e., by increasing the class of the

functions that are being compared, we obtain the exact extremum as the limit.

However, one must not use too large a value of k in the case under consideration,

inasmuch as the first-approximation formula was used for the reflection

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coefficient, and there is no advantage in computing N_k to a higher degree of reference [1].

We seek a minimum for the expression

$$J_k(a_1, a_2, \ldots a_k) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{mn} \, a_m \, a_n. \qquad (22a)$$

The extremum conditions asume the form

$$\frac{\partial J}{\partial a_i} = 2 \sum_{n=0}^{k} c_{in} a_n = 0 \quad (i = 1, 2 \ldots k). \qquad (22b)$$

We obtain a system of k equations relative to the unknowns $a_1, a_2, \ldots, a_k$

$$\sum_{n=1}^{k} c_{in} a_n = c_{i0} \, a_0 \quad (i = 1, 2, \ldots k). \qquad (22c)$$

The coefficients c_{mn} for which m + n is odd vanish. In fact, from (19) and (16) we have

$$c_{2\lambda,2\nu-1} = \frac{1}{a} \int_{-a}^{a} \int_{a}^{a} K(y-y') \cos \frac{\pi\mu}{a} y \sin \frac{\pi\nu}{a} y' \, dy \, dy'. \qquad (22d)$$

Changing variables: $\eta = -y$, $\eta' = -y'$ yields, since the kernel is even:

$$c_{2\mu,2\nu-1} = -\frac{1}{a} \int_{-a}^{a} \int_{a}^{a} K(\eta' - \eta) \cos \frac{\pi\mu}{a} \eta \sin \frac{\pi\nu}{a} \eta' \, d\eta \, d\eta' = - c_{2\mu,2\nu-1}, \qquad (22e)$$

i.e.,

т. е. $c_{2\mu,2\nu-1} = 0$ (а также $c_{2\nu-1,2\mu} = 0$, так как $c_{mn} = c_{nm}$).

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$$(22f)$$

Since c_{mn} vanishes for m + n = 2i + 1, the system (23) divides into two systems of equations:

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$$\sum_{y=1}^{x} c_{2x,2y} a_{2y} = c_{2x,0} a_0 \quad v = 1, 2, \ldots \, x; \tag{24}$$

$$\sum_{v=1}^{x} c_{2x-1,2v-1} a_{2v-1} = 0 \quad v = 1, 2, \ldots \, x \tag{25}$$

for even values of $k = 2x$.

It follows from system (25) that $a_{2v-1} = 0$, i.e., $N(y)$ is an even function of y. To determine a_{2v}, it is necessary to solve the system (24). Let us perform the computations for $k = 2$ ($x = 1$). Equation (24) yields

$$a_2 = \frac{c_{20}}{c_{22}} a_0, \tag{26}$$

where

$$c_{20} = \frac{1}{a\sqrt{2}} \int_{-a}^{a} \int_{-a}^{a} K(y-y') \cos \frac{\pi}{a} y \, dy \, dy', \tag{26a}$$

$$c_{22} = \frac{1}{a} \int_{-a}^{a} \int_{-a}^{a} K(y-y') \cos \frac{\pi}{a} y \cos \frac{\pi}{a} y' \, dy \, dy'. \tag{26b}$$

From equation (22) we obtain

$$N_1(y) = \frac{a_0}{\sqrt{2a}} (1 + \sqrt{2} \frac{c_{20}}{c_{00}} \cos \frac{\pi}{a} y). \tag{26c}$$

Returning to the variable $x = y + a$ we get

$$N(x) = N_1(x-a) = \frac{a_0}{\sqrt{2a}} \left(1 - \sqrt{2} \frac{c_{20}}{c_{00}} \cos \frac{\pi}{a} x \right). \tag{27}$$

Assume that $\omega_1 \rightarrow 0$ and $\omega_2 \rightarrow 1$. Then $c_{20} \rightarrow 0$, and c_{22} remains finite.

Equation (27) yields

$$\rho(x) = \rho_2 \left(\frac{\rho_1}{\rho_2} \right)^{\frac{x}{T}}. \tag{27a}$$

Thus the best transition piece for an operating frequency range between

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0 and infinity would be an exponential line.

If ρ_1 / ρ_2 is a complex quantity, the above method is applicable with the following modification, that instead of (22) it is necessary to use

$$N_k = \sum_{n=0}^{\kappa} (a_n + ib_n)\, \varphi_n(y).\tag{27b}$$

Corresponding to the increase in the number of unknown coefficients is the increase in the number of equations:

$$\frac{\partial J_\kappa}{\partial a_i}=0;\ \frac{\partial J_\kappa}{\partial b_i}=0\quad (i=1,2,\ \ldots\ k),\tag{27c}$$

where

$$J_\kappa = \sum_{m=0}^{\kappa} \sum_{n=0}^{\kappa} c_{mn}(a_m a_n + b_m b_n).\tag{27d}$$

Remark 1. The minimum of expression (14), under condition (13), can also be derived with a variational method.

Again we restrict ourselves to the case of real ρ_1 / ρ_2 and take the variation of (14) under condition (13); we obtain

$$\int_{-a}^{a}\int_{-a}^{a} [\delta N(y)\, N(y') + N(y)\,\delta N(y')]\, K(y-y')\, dy\, dy' + \lambda \int_{-a}^{a} \delta N(y)\, dy = 0,\tag{27e}$$

where λ is the Lagrangian multiplier.

Interchanging $y = y'$ and $y' = y$ in the second term inside the square bracket, and making use of the symmetry of the kernel K, we obtain

$$\int_{-a}^{a}\left[\int_{-a}^{a} K(y-y')\, N(y')\, dy' + \lambda\right]\delta N(y)\, dy = 0.\tag{27f}$$

Hence

$$\int_{-a}^{a} K(y-y')\, N(y')\, dy' + \lambda(y) = 0,\tag{27g}$$

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where

$$\lambda(y) = \text{const.} \qquad \text{at} \qquad -a \leq y \leq a$$

The solution to this integral equation yields the optimum function N(y); this solution can at least be obtained with the method of successive approximations.

Remark 2. In some cases it is essential to have not only the minimum reflected power, but it is also important to have a low value for the maximum reflection coefficient $|p(\omega)_{max}|$ in the prescribed frequency band $\omega_1 \leq \omega \leq \omega_2$. It is therefore possible to phrase the entire problem differently, and namely to seek such a transition piece in which $|p(\omega)_{max}|$ ($\omega_1 \leq \omega < \omega_2$) has the minimum value. However, this makes it difficult to find the optimum transition analytically. On the other hand, if the proposed method is used, there will be no anomalies in the function $|p(\omega)|$ ($\omega_1 < \omega < \omega_2$) if there are no peaks among the a_n's (n = 1, 2......k), i.e., if some of the a_n's do not differ radically from the remaining ones. If we see upon computing a_n that there are no peaks among the a_n's, we can be confident that there will be no large values of $|p(\omega)_{max}|$ ($\omega_1 < \omega < \omega_2$).

Let us note that there can be no greater peak of $|p(\omega)_{max}|$ than the value $|p(0)|$ as follows from equation (26) of reference (1).

$$|p(\omega)| < |p(0)| = \frac{1}{2} \ln \frac{\rho_1}{\rho_2}. \tag{27h}$$

On the other hand, the peak at zero cannot be eliminated and is independent of the form of the transition. The question of secondary peaks should therefore not become acute if they are smaller than the certainly-present peak at zero frequency.

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CONCLUSION

1. The external parameters a, b, c, and d of the four-terminal network that is equivalent to a non-uniform line can be obtained with a briefer method than that proposed by A. L. Fel'dshteyn[3]; with this, the Riccati differential equation is used for the reflection coefficient. It becomes possible to break up this equation into four simple equations for the unknown external parameters, and then change to the integral equations obtained by A. L. Fel' dshteyn[3].

2. The problem of optimum transition between two uniform lines with different characteristic impedances was considered by V. A. Il'in[2] and A. L. Fel'dshteyn[3,4].

the latter reaches the conclusion that the optimum line is the one approximating most closely the probability curve(i.e., one in which $N(x) \approx N_0 e^{-ax^2}$).

A criterion of the line being an optimum one, as selected in references (3, 4), is the concentration of the spectral function of $N(x)$ in the vicinity of $\omega = 0$.

The optimum-line criterion used in this article is minimum power reflection in the prescribed frequency band(ω_1, ω_2). Such a condition appears to be more rational. The use of this criterion makes it possible to obtain general expressions with which to determine the optimum function for constructing the non-uniform line that matches two uniform lines with different characteristic impedances.

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(active member, ... the society)

Considered in this article are several problems associated with computation of waveform distortion of a pulse signal passing through a minimum-phase linear system, when only the frequency characteristics of the latter are known. The waveform-distortion coefficient of the signal as a whole and the time shift of the signal are used to provide a quantitative measure of such distortion. It is shown at the same time that the widening of the pass band of typical frequency characteristics is associated with an automatic widening of the linearity band of the phase characteristic.

1. INTRODUCTION

The distortions in a signal passing through a linear system result from the fact that no practical radio circuit has ideal frequency and phase characteristics.

As long as one is merely interested in the faithful reproduction of the frequency spectrum of the signal, the phase characteristics of the circuits need not be taken into account. This is applicable to a wide extent in radio-telephone transmission systems. However, for transmission types in which it is important as a whole (television, radar, etc.) it is essential to take also the phase distortion into account.

The distortion in a linear system must thus be computed taking both the frequency and phase characteristics into account. However, there exists a large class of linear radio circuits--the so-called minimum-phase networks-- for which there is a unique relationship between the frequency and phase characteristics. The determination of distortion in such networks reduces to a mere calculation of the form of the frequency characteristic, for once the frequency characteristic is given, the phase characteristic can be obtained from it automatically. As it therefore becomes possible to select the band-

width of a system intended for reproduction of pulse signals of various forms in such a way, that the signal distortion does not exceed the prescribed limits.

It has been established that the rectangular frequency characteristic and the linear phase characteristic, usually taken as ideal, are not compatible with each other, nor can either be individually realized. We are therefore introducing in this work a standard frequency characteristic (with a phase characteristic corresponding to it), and a standard phase characteristic (with a corresponding frequency characteristic); both are realizable in actual radio networks.

To obtain a quantitative measure of distortion, we shall develop below the total-squared-error criterion, proposed by A. A. Kharkevich[1].

A detailed development of this criterion is used as a basis for investigating the passage of pulses of various shapes through minimum-phase radio networks, using two types of frequency characteristics. The bandwidth required for low-distortion reproduction of the signal waveform is established. It is also shown that the widening of the band of the standard frequency characteristic of the system is associated with an automatic improvement of its phase characteristic. Along with the development of the total-squared-error criterion, a new definition is introduced for the time delay of a signal; this definition corresponds to the physical nature of the frequency group delay.

The fact that we restrict ourselves to minimum-phase networks is of little substance, inasmuch as most practical radio networks are of the minimum-phase type.

In the following derivations, all intermediate computations are omitted and only the final results are given.

2. TYPICAL PHASE AND FREQUENCY CHARACTERISTICS

A linear system is of the minimum-phase type if its transfer function K(p) has neither poles nor zeros in the right half plane of the complex variable p.

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In minimum-phase networks there exists a unique relationship between the modulus $C(\omega)$ of the transfer function, i.e., the frequency characteristic of the network, and its argument $\varphi(\omega)$, i. e., the phase characteristic. This relationship is [2]:

$$\varphi(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\ln C(x)}{\omega - x}\, dx, \qquad (1)$$

$$\ln \frac{C(\omega)}{C(\omega_0)} = \frac{1}{\pi} \int_{-\infty}^{\infty} \varphi(x) \left(\frac{1}{x - \omega} - \frac{1}{x - \omega_0} \right) dx. \qquad (2)$$

It follows from equations (1) and (2) that a parallel shift of the frequency characteristic along the abscissa axis results in an equal parallel shift of the phase characteristic, and vice versa. We shall assume from now on that the origin of the frequency and phase characteristic has been shifted to the resonance point $\omega = \omega_0$.

A rectangular frequency characteristic and a straight-line phase characteristic, usually taken as ideal, are not only incompatible with each other, but in addition, if either one is prescribed for a minimum-phase circuit, the other is not obtainable from it. This can readily be verified if an attempt is made to use equations (1) and (2) to compute the phase (or frequency) characteristic corresponding to such an ideal frequency (or phase) characteristic.

Given below are new standard characteristics which, unlike the above-mentioned ideal characteristics, are realizable in principle in radio networks. We shall adopt as such a standard characteristic the phase characteristic shown in fig. 1, whose equation can be written in the form:

$$\varphi_1(\omega) = \begin{matrix} \kappa \omega & \omega < \Delta \\ \kappa \Delta & \omega > \Delta \end{matrix} \Bigg\} \qquad (3)$$

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It follows from equation (2) that the following frequency characteristic should correspond to such a standard phase characteristic:

$$C_1(\omega) = C_0 \left\{ \left| 1 - \frac{\omega^2}{\Delta^2} \right| \left(\frac{1 + \frac{\omega}{\Delta}}{\left| 1 - \frac{\omega}{\Delta} \right|} \right)^{\frac{\omega}{\Delta}} \right\}^{-\frac{n}{2}} \tag{4}$$

where n denotes

$$n = \frac{2}{\pi} \kappa \Delta. \tag{5}$$

Fig. 2 shows the frequency characteristic plotted from equation (4) When n = 1 the standard phase characteristic approaches the phase characteristic of a resonant amplifier with a single oscillating circuit, and when n = 2 it approaches the characteristic of a double-tuned resonant amplifier.

Assume that κ and Δ are large, but

$$\frac{\kappa}{\Delta} = \pi a^2 = \text{const}; \tag{6}$$

then $n \gg 1$, corresponding to a tuned amplifier with a large number of stages. It is evident from (4) that for this case the amplification factor will be negligibly small for $\omega > \Delta$. For $\omega > \Delta$, i.e., in the band where the phase varies linearly, equation (4) can be rewritten for the case under consideration as follows:

$$C_1(\omega) = C_0 e^{-a\omega} \sum_{p=1}^{\infty} \frac{1}{p(2p-1)} \left(\frac{\omega}{\Delta} \right)^{2p-2}. \tag{7}$$
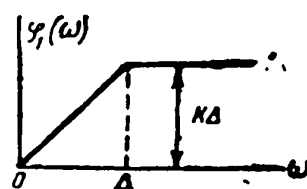
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$\varphi_1(\omega)$

$K\Delta$

0 Δ ω

Figure 1

c_0

$c_1(\omega)$

$\dfrac{c_0}{2^n}$

0 Δ ω

Figure 2

$c_2^*(\omega)$

0 Δ ω
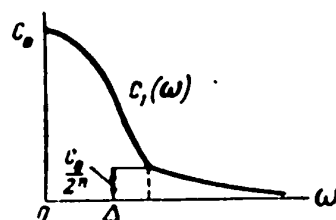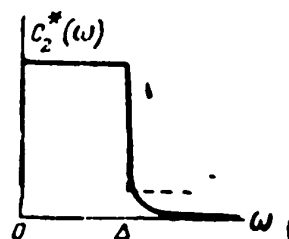
Figure 3
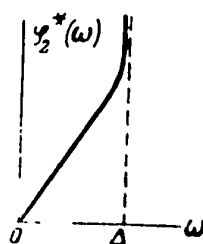
$\varphi_2^*(\omega)$

0 Δ ω

Figure 4

STAT

It is evident from eq. (7) that the frequency characteristic can be represented with sufficient accuracy by the Gaussian probability-distribution curve. The deviation of this frequency characteristic from a Gaussian one will be less than 1% up to $\omega = 0.85\,\Delta$. At the same time extending — the section where the phase characteristic (3) varies linearly—produces an increase in the bandwidth of its corresponding frequency characteristic(7).

Let us now turn to a consideration of a typical frequency characteristic. We shall consider first a broken-line /step type/ frequency characteristic (shown dotted in fig. 3), whose equation can be written in the following form:

$$C_2(\omega) = \begin{cases} e^{\alpha N} & \omega < \Delta,\ N > 0 \\ 1 & \omega > \Delta \end{cases} \qquad (8)$$

It follows from equation (1) that the following phase characteristic chould correspond to the above frequency characteristic:

$$\varphi_2(\omega) = \begin{cases} N\,\text{Arth}\,\dfrac{\omega}{\Delta} & \omega < \Delta, \\ N\,\text{Arth}\,\dfrac{\Delta}{\omega} & \omega > \Delta. \end{cases} \qquad (9)$$

A standard frequency characteristic that approximates the ideal rectangular characteristic can be obtained from the frequency characteristic $C_2(\omega)$ by subjecting the latter to a deformation as a result of multiplying $C_2(\omega)$ by the function,

$$f(\omega) = \begin{cases} 1 & \omega < \Delta \\ \left(\dfrac{\Delta}{\omega}\right)^m & \omega > \Delta, \end{cases} \qquad (10)$$

whereby $m \geqslant 1$. Then the form of the frequency characteristic will remain unchanged in the frequency band $\omega > \Delta$, but its trailing edge for $\omega > \Delta$ will be as close as desired to the abscissa axis (fig.3). The equation of the phase

characteristic for such a deformation of the frequency characteristic will

assume the form:

$$\varphi_2^o(\omega) = \frac{2m}{\pi} \left[\frac{\omega}{\Delta} + \frac{\pi N}{2m} \text{Arth} \frac{\omega}{\Delta} + \sum_{\kappa=0}^{\infty} \frac{1}{(2\kappa+1)^2} \left(\frac{\omega}{\Delta} \right)^{2\kappa+1} \right], \quad \omega < \Delta. \quad (11)$$

It is evident from eq. (11) that the linearity of the phase characteristic

$\psi_2^*(\omega)$ improves as the width Δ of its corresponding typical frequency

characteristic increases. The form of the phase characteristic corresponding

to the standard frequency characteristic is shown in fig. 4.

3. WIDENING OF THE LINEAR PORTION OF THE PHASE CHARACTERISTIC WHEN
    THE PASS BAND OF THE FREQUENCY CHARACTERISTIC IS EXTENDED. (Beginning of
    finer print in original text) - Editor.

The connection between the linearity of the phase characteristic and the

bandwidth of the frequency characteristic, established above for standard

cases, can be extended to any frequency characteristic which differs little

from the standard frequency characteristic inside the passband. Let

$$\overline{C}(\omega) = \frac{C(\omega)}{C_0} = 1 + \mu(\omega), \quad 0 < \omega < \Delta. \quad (12)$$

Then

$$\varphi(\omega) = \frac{2\omega}{\pi} \int_0^\Delta \frac{\ln(1+\mu)}{\omega^2 - x^2} dx + \frac{2\omega}{\pi} \int_\Delta^\infty \frac{\ln \overline{C}(x)}{\omega^2 - x^2} dx. \quad (13)$$

Neglecting the square of μ and denoting $\max |\mu(\omega)$ by μ_0, we obtain for

the following estimate for the first integral of the right side of (13):

$$J_1 = \frac{2\omega}{\pi} \int_0^\Delta \frac{\ln(1+\mu)}{\omega^2 - x^2} dx < \frac{2}{\pi} \mu_0 \text{Arcth} \frac{\Delta}{\omega}. \quad (14)$$

The second term of the right side of (13) is proportional to the frequency;

at the same time the integral, which is the proportionality coefficient, depends

little on the frequency for $\omega < \Delta$. Restricting ourselves to a quadratic correc-

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tion, we can represent this integral as follows:

$$J_2 = \int_\Delta^\infty \frac{\ln \overline{C}(x)}{\omega^2 - x^2}\,dx - \Delta\left(\int_0^1 \frac{\ln \overline{C}\left(\frac{\Delta}{u}\right)du}{\Delta^2 - \omega^2 u^2}\right)_{\omega \to 0} + \cdots + \frac{\omega^2 \Delta}{2}\left[\frac{d^2}{d\omega^2}\int_0^1 \frac{\ln \overline{C}\left(\frac{\Delta}{u}\right)du}{\Delta^2 - \omega^2 u^2}\right]_{\omega \to 0}. \quad (14\,a)$$

Denoting

$$S = \int_0^1 \ln \overline{C}\left(\frac{\Delta}{u}\right)du, \quad r = \frac{\int_0^1 u^2 \ln \overline{C}\left(\frac{\Delta}{u}\right)du}{\int_0^1 \ln \overline{C}\left(\frac{\Delta}{u}\right)du}. \quad (15)$$

we obtain after uncomplicated computations

$$J_2 = \frac{S}{\Delta}\left(1 + r\,\frac{\omega^2}{\Delta^2}\right). \quad (16)$$

To determine the width of the linear portion of the phase characteristic of the network, assume the frequency band to be such that the relative deviation from linearity of the phase characteristic does not exceed within this band a certain ϵ ; i.e., we have for all values of ω within the linear portion of the phase characteristic:

$$\epsilon > \frac{\varphi(\omega) - \frac{2S}{\pi\Delta}\,\omega}{\frac{2S}{\pi\Delta}\,\omega}. \quad (17)$$

It therefore follows from (14) and (16) that

$$\frac{\mu_n}{S}\cdot\frac{\Delta}{\omega}\,\mathrm{Arcth}\,\frac{\Delta}{\omega} + r\left(\frac{\omega}{\Delta}\right)^2 < \epsilon. \quad (18)$$

It follows from (18) that as the band Δ of the frequency characteristic becomes wider, it is possible to effect a proportional increase in the value of the frequencies ω , without disturbing the inequality (17). This means that increasing the pass band of the frequency characteristic is automatically accompanied by an increase in the linear portion of the phase characteristic.

Consequently, if the bandwidth of the frequency characteristic is suf-
ficiently wide, and differs little from the standard, the required linearity
band of the phase characteristic will be automatically assured, and thus
the signals will be faithfully reproduced when passing through the linear
systems. **(End of fine print in original) - Editor.**

4. DETERMINATION O THE EXTENT CF DISTORTION

To have a uniform single-valued approach to the evaluation of the waveform
distortion, it is first necessary to agree on the criterion of faithful re-
production of a signal. We shall use a power criterion, which reduces to the
evaluation of the overall deviation of the waveform of the received signal
from the form of the transmitted signal, with respect to the amount of the
total squared error[1]. The squared-error criterion can be made quite
stringent and suitable for application to many problems of signal waveform
reproduction in radio engineering.

Let the form of the envelope of the signal applied to the network be
represented by the function $u(t)$, and let the distorted form of the enve-
lope of this signal at the output of the same network be represented by the
function $e(t)$. As a measure of the distortion of the signal as it passes
through the network, we take the value of the least squared deviation of the
envelope of the distorted signal, relative to the envelope of the undistorted
signal, i.e., the integral

$$\eta(\lambda, \, t_0) = \int_0^\infty [\lambda e(t) - u(t - t_0)]^2 \, dt. \tag{19}$$

The measure of distortion, as determined by. eq. (19), depends on two
parameters, the normalizing multiplier λ and the time shift t_0, with the
latter indicating the position of the undistorted signal relative to the dis-
torted one. In conformance with the accepted definition, these parameters

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should be obtained from the condition that the function $\eta(\lambda, t_o)$ be a minimum. Applying this condition to (1°) we obtain

$$\eta = w - \frac{[\int_0^\infty e(t) u(t - t_0) dt]^2}{\int_0^\infty e^2(t) dt}, \qquad (20)$$

where ω is the power level of the signal, equal to the power delivered by the undistorted signal to a unit resistive load, i.e.

$$w = \int_0^\infty u^2(t) dt = \int_0^\infty u^2(t - t_0) dt. \qquad (21)$$

Let the following quantity be called the relative signal-distortion coefficient due to passage through the network:

$$v = \frac{\eta}{w} = 1 - \frac{[\int_0^\infty e(t) u(t - t_0) dt]^2}{w \int_0^\infty e^2(t) dt}. \qquad (22)$$

In accordance with the minimum condition $v(t_o)$, the time shift t_o must satisfy the equation

$$\frac{d}{dt_0} \int_0^\infty e(t) u(t - t_0) dt = 0. \qquad (23)$$

The quantity t_o computed from eq. (23) can be used to define the delay time of the distorted signal. Such a definition describes more accurately the physical substance of the occurring frequency group delay than does the definition adopted in the literature, usually related to the tangent of the slope of the phase characteristics at the origin.

The parameter t_o must be considered as a characteristic measure of signal distortion, supplementing the parameter v. The full description of the signal distortion is thus accomplished with two parameters: the distortion coefficient v and the time shift t_o.

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In case of a signal having a rectangular-pulse envelope of duration τ, equation (23) reduces to the following:

$$e(t_0 + \tau) = e(t_0),\qquad(24)$$

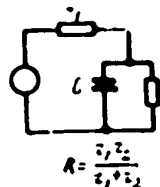For a signal with triangular envelope the form is:

FIG. 5

$$\int_{t_0}^{t_0 + \frac{\tau}{2}} e(t)\,dt = \int_{t_0 + \frac{\tau}{2}}^{t_0 + \tau} e(t)\,dt.\qquad(25)$$

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To judge the extent by which the adopted criterion is sensitive to changes in the form of the signal envelopes, let us compute the distortion coefficients in the simplest cases. When a rectangular pulse of duration τ passes through a network that produces distortion at high frequencies (fig. 5), we have

$$v = 1 - \frac{\left[1 - \frac{1}{\beta\tau}\ln(2 - e^{-\beta\tau})\right]^2}{1 - \frac{1 - e^{-\beta\tau}}{\beta\tau}};\qquad(26)$$

$$t_0 = \frac{1}{\beta}\ln(2 - e^{-\beta\tau}).\qquad(27)$$

Fig. 6 shows several pulses that are distorted after passing through the network under consideration, and indicates the corresponding value of ν as computed from eq. (26).

Let us denote by $K(i\omega)$ the complex transfer coefficient of the network, to whose input a radio pulse with envelope $u(t)$ and carrier frequency ω_0, is applied. Let the complex spectral density of the pulse envelope equal $R(i\omega)$. Then the distorted envelope of the radio pulse at the output of the network can be represented with the aid of a Fourier integral of the following form:

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$$e(t) = \int_{-\infty}^{\infty} R(i\omega) K[i(\omega+\omega_0)] e^{i\omega t} d\omega. \quad (28)$$
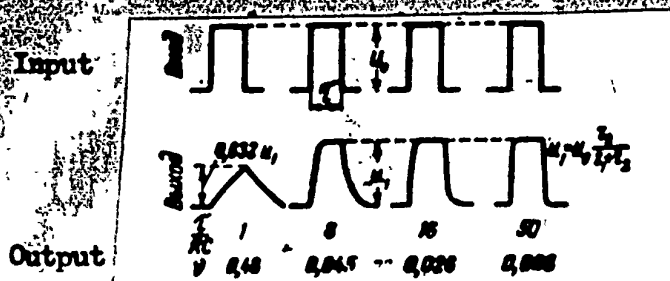

Fig. 6

It is evident from eq. (28) that the pulse envelope obtained at the output of the network is the same as that obtained when a pulse without a high-frequency content passes through a network, whose characteristic is shifted into the low-frequency region by an amount ω_o [4]. The quantity $K(i\omega)$ will therefore denote from now on the complex characteristic of the network, shifted in the indicated manner toward the low-frequency region, and the radio-frequency pulse will be replaced by a d-c pulse.

Using (28) it is possible to rewrite eq. (22) for the relative distortion coefficient in the following form:

$$\nu = 1 - \frac{\pi}{\omega} \frac{\left\{\int_{0}^{\infty} C(\omega) F^2(\omega) \cos[\varphi(\omega) - \omega t_0] d\omega\right\}^2}{\int_{0}^{\infty} C^2(\omega) F^2(\omega) d\omega}, \quad (29)$$

where $C(\omega)$ and $\varphi(\omega)$ are the frequency and phase characteristics of the network, and $F(\omega) = |R(i\omega)|$.

5. PASSAGE OF VARIOUS SIGNALS THROUGH NETWORKS WITH STANDARD CHARACTERISTICS

We shall investigate with the aid of the adopted criterion the distortion of signals with envelopes of various shapes. In the examples given here the distortion will be evaluated only by the magnitude of the parameter ν .

In those cases, when the signal is propagated without limit, it is necessary to establish arbitrarily what is meant by the duration of the signal.

Let u_0 be the maximum envelope voltage, which occurs at $t=0$ in the case of

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the signals under consideration. Let the signal duration denote the duration of such a rectangular signal with voltage u_o, whose power level equals the power level of the signal under consideration, i.e.,

$$\tau = \frac{\int_{-\infty}^{\infty} u^2(t)\,dt}{u_0^2} = \frac{w}{u_0^2}. \tag{30}$$

For a bell-shaped pulse, whose envelope is represented by the equation $u_2(t) = u_o e^{-\beta^2 t^2}$, the duration equals

$$\tau = \frac{1}{\beta} \sqrt{\frac{\pi}{2}}. \tag{31}$$

For a limited-spectrum signal, whose envelope is represented by the equation $u_3(t) = u_o \dfrac{\sin \beta t}{\beta t}$, the duration equals

$$\tau = \frac{\pi}{\beta}. \tag{32}$$

The relative distortion coefficient is computed from eq. (29) as a function of the parameter $\mu = \Delta F \tau$, i.e., the product of the bandwidth of the frequency characteristic and the duration of the signal. The width of the pass band is meant to be the integral width of the band, determined as the base of a rectangle having an altitude C_o^2 and an area $\dfrac{1}{2\pi} \int_0^{\infty} c^2(\omega)\,d\omega$.

When a series of N rectangular pulses, each of duration τ, passes through a network having a standard frequency characteristic, the equation for computing the relative distortion coefficient assumes the form:

$$v_{1N}^2 = 1 + \frac{2}{\pi^2 \mu N} \sin^2 \pi\mu \sum_{r=0}^{2m} C_{r,N} (2\cos 2\pi\mu)^2 (N - 2m + r - 1) -$$

$$- \frac{2}{\pi N} \sum_{r=0}^{2m} C_{r,N} [z_r^{(1)} - 2\sigma_r^{(2)}]. \tag{33}$$

where

$$C_{r,N} = \sum_{q=0}^{r} a_{r,N} \cdot a_{r-q,N}; \quad a_{\kappa,N} = (-1)^{\kappa} \binom{N-\kappa-1}{\kappa}; \qquad (34)$$

$$c_r^{(1)} = \sum_{p=0}^{N-2m+r-1} \binom{2(N-2m+r-1)+1}{p} [2(N-2m+r-1)-2p+1] \times$$

$$\times \text{sl} \{[2(N-2m+r-1)-2p+1] 2\pi\mu\};$$

$$c_r^{2} = \sum_{p=0}^{N-2m+r-1} \binom{2(N-2m+r-1)}{p} [2(N-2m+r-1)-2p] \times$$

$$\times \text{sl} \{2(N-2m+r-1-p) 2\pi\mu\}. \qquad (35)$$

Here $m = E\left(\dfrac{N-1}{2}\right)$ is the integral part of the number $\dfrac{N-1}{2}$, and (p/q) is the abbreviated notation for the binomial coefficient. Fig. 7 shows graphically the dependence of $v_{1n}(r)$ on μ for N = 1, 2, 3. When a bell-shaped pulse passes through a network with a standard frequency characteristic, the relative distortion coefficient is

$$v_2^{(r)} = 1 - \Phi(2\mu\sqrt{\pi}), \qquad (36)$$

where $\Phi \cdot (x)$ is Cramp's function. If the pulse is of the limited-spectrum type, the coefficient is

$$v_3^{(r)} = \begin{cases} 1 - 2\mu & \mu < 0.5 \\ 0 & \mu > 0.5 \end{cases} \qquad (37)$$
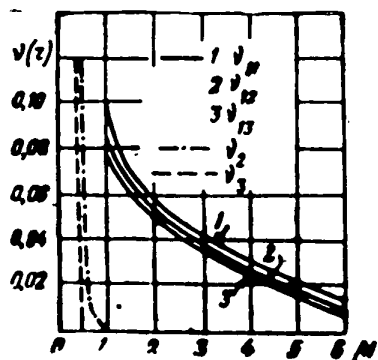

Figure 7

The graphs corresponding to eq. (36) and (37) are shown in Fig. 7. It can be seen from these graphs that if the distortion coefficient is rigidly limited to a value of 3% the required pass band of the standard characteristic is determined from the following relationships:

$$\text{rectangular pulse} \quad \Delta F > \frac{4}{\tau}, \tag{38}$$

$$\text{bell-shaped pulse} \quad \Delta F > \frac{0.6}{\tau}, \tag{39}$$

$$\text{limited-spectrum pulse} \quad \Delta F > \frac{0.5}{\tau}. \tag{40}$$

For the case of a network with a standard phase characteristic and n = 2, the distortions are characterized by the following relative coefficients:

$$v_{11}^{(\phi)} = 1 - \frac{2}{\pi} \left[si\left(\frac{\pi\mu}{0.7}\right) - \frac{1.4}{\pi\mu} \sin^2 \frac{\pi\mu}{1.4} \right] \tag{41}$$

for the passage of a rectangular pulse of duration τ, whereby $\mu > 2$;

$$v_2^{(\phi)} = 1 - \frac{\left(1 - \frac{1}{16\pi\mu^2}\right)^2}{1 - \frac{1}{8\pi\mu^2}} \, \Phi\left(\frac{\sqrt{\pi}}{0.7}\mu\right) \tag{42}$$
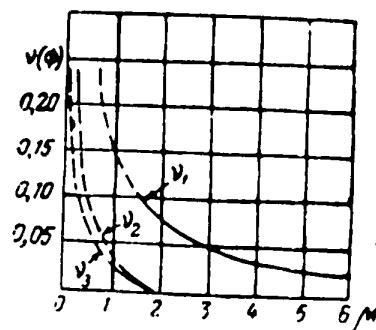

Figure 8

for the passage of a bell-shaped pulse, where $\mu > 0.7$, and

$$\nu_j^{(\Phi)} = 1 - \frac{\left[\frac{3}{2}\mu_1\left(e^{\frac{1}{2\mu_1}}-1\right)+\frac{1}{4}\left(1-\frac{1}{4\mu_1}\right)e^{\frac{1}{2\mu_1}}\right]^2}{\frac{5}{8}\mu_1\left(e^{\frac{1}{\mu_1}}-1\right)+\frac{3}{8}\left(1-\frac{1}{2\mu_1}\right)e^{\frac{1}{\mu_1}}} \tag{43}$$

for the passage of a limited-spectrum pulse, whereby $\mu = 0.7$ $\mu_1 > 0.7$.

Equations (41) to (43) are approximate. The limitations imposed in these equations on the parameter μ insure sufficient accuracy for practical purposes. The dependence of the distortion coefficient on the parameter μ for the case under consideration is presented in the form of suitable graphs in Fig. 8. The computed curves are extended with dotted lines into the regions where the equations given for the computation of $\nu(\Phi)$ are no longer sufficiently accurate. The true values of the coefficients for these regions lie above these dotted curves.

From the graphs presented it is evident that for a permissible 3% distortion the pass band of the frequency characteristic, corresponding to the standard phase characteristic, must be determined from the following relationships:

$$\text{rectangular pulse} \quad \Delta F > \frac{4}{\tau}, \tag{44}$$

$$\text{bell-shaped pulse} \quad \Delta F > \frac{1.2}{\tau}, \tag{45}$$

$$\text{limited-spectrum pulse} \quad \Delta F > \frac{1}{\tau}. \tag{46}$$

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CONCLUSION

1. Most practical radio networks are of the minimum-phase type, and consequently their phase characteristic can be uniquely determined from the

frequency characteristic. The distortion of the signals passing through such networks can therefore be computed from the frequency characteristic alone.

2. Increasing the pass band of a frequency characteristic that differs little from the standard one causes an automatic increase in the linearity region of the phase characteristic. The pass band of the frequency characteristic can always be increased in such a way that the phase distortion can be disregarded.

3. Using the total-square-error criterion as a measure of the distortion, we find a new definition for the time delay of the signal; this definition reflects more accurately the physical nature of the phenomenon.

4. Inasmuch as the usually employed ideal frequency and phase characteristics are incompatible with each other, nor can either one be individually realized, it is best to introduce the standard frequency and phase characteristics proposed in this article.

5. For certain signal waveshapes, this article gives relationships with the aid of which it is possible to determine the pass bands of linear systems with standard characteristics required for undistorted reproduction of the signal.

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ERRORS IN MEASURING FREQUENCY CHARACTERISTICS BY FREQUENCY-MODULATION METHODS

I. T. TURBOVICH

active member of the society

The article proposes a method for determining the errors in measuring the frequency characteristics of four-terminal networks by the frequency-modulation method. It is shown that this error can be divided into two components. Simple formulas are given for computing these components.

During recent years there has been an ever-increasing development of methods for rapidly measuring the frequency characteristics of various electric systems, whereby these characteristics are presented on the screen of the oscillograph. However, because of the settling-down processes that result, the frequency characteristics differ from those plotted point-by-point.

The literature contains an analysis of the question of the action of frequency-modulated oscillations on oscillating circuits with one [1,2] or two [3] degrees of freedom. This question is discussed in greater detail in reference [1], which also contains a thorough review of the literature.

In the above works, the investigation of the effect of frequency-modulated oscillations even on such simple oscillating circuits leads to complicated expressions, containing functions that have not yet been tabulated. This naturally makes the use of the derived expressions difficult. Reference [1] contains a dynamic resonance curve of a tuned circuit for the case of a rather rapid change in the frequency of the applied voltage; this curve is obtained by computation, and differs radically in its nature from the static resonance curve.

The question of the effect of frequency-modulated oscillations on more complex oscillating systems is not covered in the literature, as far as we know.

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development of these methods.

Of great significance in the design and use of instruments for rapidly measuring frequency characteristics is an evaluation of the errors in measurement results. Of practical interest here is only the evaluation of small errors, for once large errors appear the measurement loses its meaning.

The author of this article has undertaken the task of determining the errors in measuring frequency characteristics, assuming beforehand that these errors are small. The problem can be formulated in the following manner:

A frequency-modulated voltage is applied to the investigated(passive) four-terminal network. The dynamic frequency characteristic of the transfer coefficient is defined as the ratio of the output voltage of the four-terminal network to its input voltage. The dynamic transfer coefficient differs from the static error by the value of the error, which must be determined.

Let the voltage u_1 be applied to the input of the four-terminal network at the instant t=0, and let its frequency vary during the time $0 \leqslant t \leqslant T$ linearly from ω_1 to ω_2, i.e.,

$$\omega(t) = \omega_1 + \frac{\omega_2 - \omega_1}{T} t = \omega_1 + \lambda t, \qquad (1)$$

where

$$\lambda = \frac{\omega_2 - \omega_1}{T} \qquad (2)$$

is the rate of change of frequency.

The frequency-modulated voltage at the input of the four-terminal network can be written as

$$u_1 = e^{i \varphi(t)}, \qquad (3)$$

where

$$\varphi(t) = \int_0^t (\omega_1 + \lambda t) dt = \omega_1 t + \frac{\lambda t^2}{2}. \tag{4}$$

Assume that the measured four-terminal network has a frequency characteristic:

$$S(\omega) = \int_0^\infty f(\tau) e^{-i\omega\tau} d\tau, \tag{5}$$

where $f(\tau)$ is the response of the four-terminal network to a unit pulse applied at its input.

Applying the convolution theorem, it is possible to obtain an expression for the output voltage u_2 of the four-terminal network, in the form

$$u_2 = \int_0^t f(\tau) e^{i\varphi(t-\tau)} d\tau. \tag{6}$$

Substituting expression (4) into (6) we obtain after transformation:

$$u_2 = e^{i\varphi(t)} \int_0^t f(\tau) e^{-i\omega\tau} e^{i\frac{\lambda\tau^2}{2}} d\tau. \tag{7}$$

The integral in expression (7) represents the ratio of the output to input voltages in the four-terminal network, i.e., the dynamic transfer function $S_\partial(\omega)$:

$$S_\partial(\omega) = \int_0^t f(\tau) e^{-i\omega\tau} e^{i\frac{\lambda\tau^2}{2}} d\tau. \tag{8}$$

The magnitude $\Delta S(\omega)$ of the error is defined as the difference between the static and dynamic transfer coefficients, $S(\omega)$ and $S_\partial(\omega)$:

$$\Delta S(\omega) = S(\omega) - S_0(\omega) = \int_0^\infty f(\tau)e^{-i\omega\tau}d\tau - \int_0^t f(\tau)e^{-i\omega\tau}e^{i\frac{\lambda\tau^2}{2}}d\tau =$$

$$= \int_t^\infty f(\tau)e^{-i\omega\tau}d\tau + \int_0^t f(\tau)e^{-i\omega\tau}(1 - e^{i\frac{\lambda\tau^2}{2}})d\tau =$$

$$= \Delta_t S(\omega) + \Delta_\lambda S(\omega). \tag{9}$$

Thus, the overall error can be represented in the form of two components:

$$\int_t^\infty f(\tau)e^{-i\omega\tau}d\tau = \Delta_t S(\omega), \tag{10}$$

i.e., the error due to the settling-down process occurring when a voltage with constant frequency ω is applied to the input of the four terminal network at the instant $t = 0$, and a component,

$$\int_0^t f(\tau)e^{-i\omega\tau}(1 - e^{i\frac{\lambda\tau^2}{2}})d\tau = \Delta_\lambda S(\omega), \tag{11}$$

i.e., the error due to the settling-down process due to the frequency changing at a rate $\lambda = \dfrac{\omega_2 - \omega_1}{T}$.

In accordance with the previously-assumed condition, the modulus of either error is small compared with the maximum modulus of the transfer coefficient $F_{max}(\omega) = |S(\omega)|_{max}$:

$$|\Delta_t S(\omega)| \ll F_{max}(\omega), \tag{12}$$

$$|\Delta_\lambda S(\omega)| \ll F_{max}(\omega). \tag{13}$$

Let us consider the first error $\Delta_t S(\omega)$. This error appears open switching at the instant t=0 and is at first not small, but decreases with time and drops to within the permissible value starting with the instant t_1.

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Consequently it is impossible to use a certain portion at the start of the instrument scale (where $t < t_1$). The magnitude of this portion can be determined if the error $\Delta_t S(\omega)$ is known as a function of time.

Assume that $S(\omega)$ can be represented as a sum of simple fractions:

$$S(\omega) = \sum_\kappa \frac{a_\kappa}{\delta_\kappa - I(\omega_\kappa - \omega)}. \tag{14}$$

In this case the response $f(t)$ of the system to a unit pulse can be written in the form:

$$f(t) = \sum_\kappa a_\kappa e^{(I\omega_\kappa - \delta_\kappa)t}. \tag{15}$$

Substituting (15) into expression (10) for $\Delta_t S(\omega)$ we get

$$\Delta_t S(\omega) = \sum_\kappa \frac{a_\kappa e^{[I(\omega_\kappa - \omega) - \delta_\kappa]t}}{\delta_\kappa - I(\omega_\kappa - \omega)}. \tag{16}$$

Since the modulus of the sum is less than or equal to the sum of the moduli,

$$|\Delta_t S(\omega)| < \sum_\kappa \frac{|a_\kappa| e^{-\delta_\kappa t}}{\sqrt{\delta_\kappa^2 + (\omega_\kappa - \omega)^2}}. \tag{17}$$

The factors in the form $e^{-\delta_k t}$ in expression (17) indicate that the error $\Delta_t S(\omega)$ decreases in time.

Let us turn to consider the second error $\Delta_\lambda S(\omega)$. This error is a fundamental one, for it is due to the settling-down process occurring as a result of changing the frequency, and takes place at any part of the instrument scale.

The condition (13) that this error is small for all values of ω can be satisfied only if the rate, λ, of the frequency change is sufficiently small to have the following condition satisfied at all values of τ for

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which the damping function $f(\tau)$ affects the magnitude of the integral:

$$|1 - e^{i\frac{\lambda\tau^2}{2}}| \approx \left|-\frac{i\lambda\tau^2}{2}\right| \ll 1. \tag{18}$$

Inserting (18) into eq. (11) for $\Delta_\chi S(\omega)$, we get

$$\Delta_\lambda S(\omega) = -i\frac{\lambda}{2}\int_0^t f(\tau)e^{-i\omega\tau}\tau^2 d\tau = \frac{i\lambda}{2}\frac{d^2}{d\omega^2}\int_0^t f(\tau)e^{-i\omega\tau}d\tau. \tag{19}$$

The last integral in (19) differs from eq. (5) for the static transfer coefficient $S(\omega)$ by a small quantity $\Delta_t S(\omega)$. Consequently it is possible to write, to an accuracy within second-order small quantities, a final expression for the error of the complex transfer coefficient $\Delta_t S(\omega)$.

$$\Delta_\lambda S(\omega) = \frac{i\lambda}{2}\frac{d^2 S(\omega)}{d\omega^2}. \tag{20}$$

Using expression (20), it is possible to determine the error $\Delta_\lambda \Phi(\omega)$ in the modulus of the transfer coefficient, and the error $\Delta_\lambda \varphi(\omega)$ in its phase. These errors are interconnected by the following well-known relationship:

$$\frac{\Gamma \Delta_\lambda S(\omega)}{S(\omega)} = \frac{\Delta_\lambda \Phi(\omega)}{\Phi(\omega)} + i\Delta_\lambda \varphi(\omega). \tag{21}$$

It is easy to obtain from (20) and (21) an expression for the relative error in the modulus of the frequency characteristic of the transfer coefficient:

$$\frac{\Delta_\lambda \Phi(\omega)}{\Phi(\omega)} = -\frac{\lambda}{2}\operatorname{Im}\left[\frac{1}{S(\omega)}\frac{d^2 S(\omega)}{d\omega^2}\right] \tag{22}$$

and an expression for the phase error of the transfer coefficient:

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$$\Delta_\lambda \varphi(\omega) = \frac{\lambda}{2} \operatorname{Re}\left[\frac{1}{S(\omega)} \frac{d^2 S(\omega)}{d\omega^2}\right]. \tag{23}$$

Using eq. (20), (22), and (23) it is possible to determine the error in the complex transfer coefficient, due to the settling down process resulting from the frequency changing at a rate λ, and also the errors in the modulus and phase of the transfer coefficient of any passive four-terminal network.

It is also interesting to find the shift along the frequency axis, relative to the static characteristics, produced in the dynamic characteristics of the modulus and phase of the transfer coefficient.

Using the well-known relationships:

$$\Delta\Phi(\omega) = \Delta\omega \frac{d\Phi(\omega)}{d(\omega)} = \Delta\omega \operatorname{Re}\left[\frac{1}{S(\omega)} \frac{dS(\omega)}{d\omega}\right], \tag{24}$$

$$\Delta\varphi(\omega) = \Delta\omega \frac{d\varphi(\omega)}{d\omega} = \Delta\omega \operatorname{Im}\left[\frac{1}{S(\omega)} \frac{dS(\omega)}{d\omega}\right] \tag{25}$$

and eq. (22) and (23) derived above, it is possible to obtain the following computation formulas:

$$\Delta_1 \omega = -\frac{\lambda}{2} \frac{\operatorname{Im}\left[\frac{1}{S(\omega)} \frac{d^2 S(\omega)}{d\omega^2}\right]}{\operatorname{Re}\left[\frac{1}{S(\omega)} \frac{dS(\omega)}{d\omega}\right]} \tag{26}$$

with

$$\text{при } \Phi_\partial(\omega - \Delta_1 \omega) = \Phi(\omega)$$

and

$$\Delta_2 \omega = \frac{\lambda}{2} \frac{\operatorname{Re}\left[\frac{1}{S(\omega)} \frac{d^2 S(\omega)}{d\omega^2}\right]}{\operatorname{Im}\left[\frac{1}{S(\omega)} \frac{dS(\omega)}{d\omega}\right]} \tag{27}$$

with

$$\varphi_\partial(\omega - \Delta_2 \omega) = \varphi(\omega).$$

From eq. (26) it is possible to determine the shift of the dynamic characteristic $\Phi_\partial(\omega)$ of the modulus of the transfer coefficient relative to the static

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characteristic $\Phi(\omega)$, and from eq. (27) it is possible to determine the shift of the dynamic characteristic $\varphi_{\partial}(\omega)$ of the phase of. the transfer coefficient relative to the static characteristic $\varphi(\omega)$.

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STUDY OF A SELF-EXCITED CRYSTAL OSCILLATOR
EMPLOYING THE SHEMBEL' CIRCUIT

by

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Expressions are obtained for design of self-excited quartz-crystal oscillators employing the Shembel' circuit. A method is indicated for engineering design of the self-excited oscillator, using the piecewise-linear idealization of the tube characteristic. The calculation data are compared with these obtained experimentally. The phenomenon of oscillatory hysteresis was observed.

1. INTRODUCTION

One of the variants of a self-excited crystal oscillator employing the Shembel circuit is shown in Fig. 1.
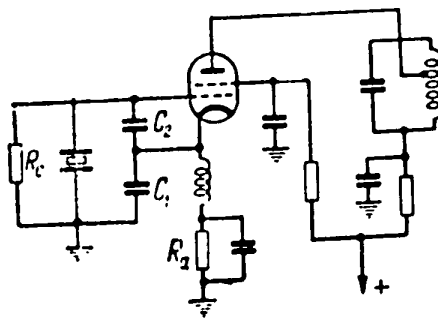

Figure 1

This circuit consists of two parts -- internal and external. The internal part is a capacitive delta circuit whose elements are the anode coupling capacitor C_1, feedback capacitor C_2, and the quartz crystal.

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Resistance R_g provides a path for the d-c component of the grid current.
When grid current flows, a certain automatic bias is produced across R_g. In
addition, automatic bias is also produced across resistance R_a by the d-c com-
ponent of the emission current. The resistance R_a is selected such as not
to exceed the permissible power dissipation in the anode or screen grid when
the oscillations are interrupted.

To reduce the load reaction on the inner part of the circuit, the outer
tank circuit is tuned to the third or second harmonic of the place current.

The above circuit is widely used; its parameters are usually chosen
experimentally, for there are no simple equations for their computation.
It is the purpose of this article to provide the simplest possible relation-
ships for computing the stationary characteristics of the self-excited osc-
illator using the Shembel' circuit, and to show the extent to which these
relationships agree with experiment.

2. DERIVATION OF INITIAL EQUATIONS

To derive the equations that describe the behavior of the internal part
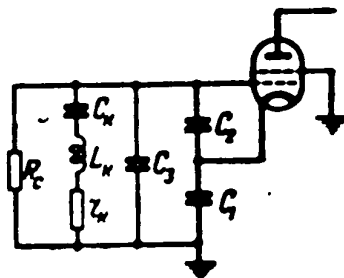of the circuit, let us consider Fig. 2.

Рис. 2

Shown here is only part of the circuit of fig. 1, with the quartz resonator
being replaced by an equivalent electric circuit, and with the circuits that
carry no a-c components omitted. Capacitance C_3 of fig. 2 represents the

total capacitance, comprising the static capacitance of the quartz, the capacitance of the crystal holder, and the parasitic circuit capacitances between the control and screen grids. Capacitances C_1 and C_2 take into account the parasitic capacitances of the corresponding parts of the circuit. Resistance R_g was left in fig.2 to take into account its shunting effect on the crystal.

The circuit of fig. 2 is not suitable for study, since it leads to rather cumbersome computations. The computations can be considerably simplified if the three delta-connected capacitances C_1, C_2, and C_3 are replaced by the three equivalent star-connected capacitances C_a, C_g, and C_q (fig.3). The following equations give the relationships between the capacitances of fig. 2 and those of fig. 3:

$$C_a = \frac{\Sigma CC}{C_3}; \quad C_g = \frac{\Sigma CC}{C_1}; \quad C_q = \frac{\Sigma CC}{C_2}, \tag{1}$$

where

$$\Sigma CC = C_1 C_2 + C_2 C_3 + C_3 C_1. \tag{1a}$$

The computation equations can be simplified still further if the shunting effect of the self-bias resistor R_g on the self-oscillation frequency is approximated by an apparent increase in the resistance r_q produced by an additional resistance

$$r' = r_q + r_{aq}. \tag{2}$$

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where

$$r_{ok} = \frac{1}{(\omega C_{ok})^2 R_c}, \qquad (3)$$

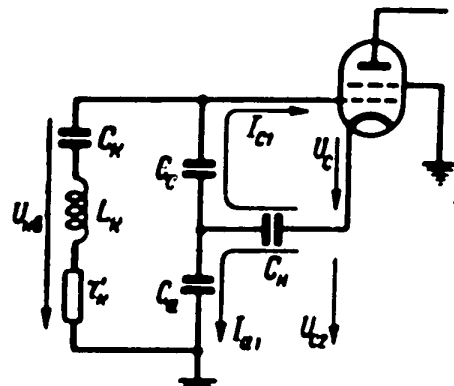$$C_{ok} = C_2 + \frac{C_1 C_3}{C_1 + C_3}. \qquad (4)$$

Puc. 3

After performing the above transformation, we obtain in fig. 3 an equivalent circuit which will be used to derive the initial equations.

Operating experience with such oscillators showed that the following must be taken into account in their design:

On one hand, to obtain enough voltage across the external circuit it is desirable that the amplitude of the grid voltage U_g be greater; on the other hand, to reduce the load on the crystal, its voltage U_q should be less. It can be assumed roughly that $U_q \approx U_g + U_{g2}$, where U_{g2} is the amplitude of the screen-grid voltage relative to the cathode. It follows therefore that if the value of U_g is prescribed, it is desirable to have a lower value of U_{g2}. In addition, a low value of U_{g2} is also desirable to avoid large screen-grid current, i.e., to avoid overvoltage operation of the control grid.

For the reasons stated, the capacitances C_1 and C_2 are chosen such as to make U_g considerably greater than U_{g2}; then the order of magnitude of the feedback coefficient $k = U_g/U_{g2}$ becomes only a few times greater than unity. A large value of k produces a peculiarity in the analysis of this circuit. If k is large it is possible to ignore the reaction of the screen voltage on the emission and grid currents, but it is important to take into account the reaction of the control grid current on the internal circuit. (By emission current is meant the sum of the plate and screen-grid currents.)

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In deriving the equations we shall assume that the self-oscillation frequency ω is near the resonant frequency ω_0 of the circuit, consisting of capacitances C_q, C_g and C_p and inductance I_q (fig. 3). This frequency can be expressed in term of the natural frequency ω_q of the crystal where C_{eg} is determined from eq. (').

$$\omega_0 = \omega_\kappa \sqrt{1 + \frac{C_\kappa}{C_{ai}}}. \tag{5}$$

Inasmuch as the equivalent capacitance C_q of the crystal is quite small compared with C_{eg}, the frequency ω_0 approximates the frequency ω_q.

For the sake of abbreviation, we shall denote the symbolic impedances of the arms of the internal circuit, shown in fig. 3, as

$$Z_a = -i\,x_a; \quad Z_c = -i\,x_c; \quad Z_\kappa = -i\,x_\kappa. \tag{6}$$

where

$$x_a = \frac{1}{\omega_0 C_a}, \quad x_c = \frac{1}{\omega_0 C_c}, \quad x_\kappa = \frac{1}{\omega_0 C_\kappa}. \tag{6'}$$

The self-oscillation frequency ω is replaced here by the resonant frequency of the tank circuit ω_0, inasmuch as the relative discrepancy between the two is quite small.

Let us also introduce an expression for the complex impedance measured around the closed circuit of fig. 3.

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$$Z_0 = r_\kappa'(1 + i\alpha), \tag{7}$$

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in which are generalized the detuning

$$\alpha = \frac{2(\omega - \omega_0)}{\omega_0 \delta},$$

(7a)

and the damping of the circuit

$$\delta = \frac{r_k'}{\omega_0 L_k}.$$

(7b)

We assume that the Shembel' circuit employs a tetrode and that the resultant shielding factor of the two grids is small. We therefore do not take into account the effect of the plate voltage on the emission current. Furthermore, as discussed above, it is possible to ignore the effect of the screen grid voltage on the emission and grid currents. This means that the harmonics of the emission and grid currents depend only on the amplitude of grid voltage U_g and on the bias voltage E_g. Consequently, to describe the behavior of the internal portion of the self-oscillator, the following two equations are of interest: the first relates the amplitude of the fundamental components of currents I_1 and I_{g1} with the amplitude of the grid voltage U_g, and the second relates the d-c components of currents I_0 and I_{g0} with the bias voltage E_g. If the time constants of the self-bias circuits are considerable less than the time constant of the tuned circuit with the crystal, it is possible to assume that the second equation will be identical with the equation for stationary operating conditions. This equation will be considered later; for the time being we turn to the first equation.

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The arrows in Fig. 3 show the positive polarities of the currents and voltages. The equation we are interested in can be derived most simply by the superposition method. Computing first the individual grid voltages produced separately by the emission and grid currents, and combining the two, we obtain: (complex quantities will be denoted from now on with a bar above the symbol).

$$\overline{U}_e = -\frac{Z_c Z_a}{Z_a}\left(\overline{I}_1 - \frac{Z_c}{Z_a}\overline{I}_{c1}\right) - Z_H\left[\overline{I}_1 + \left(1 + \frac{Z_c}{Z_H}\right)\overline{I}_{c1}\right]. \qquad (8)$$

Rewriting the complex impedances in accordance with (6) and (7), we transform this equation as follows:

$$(1 + i\alpha)\,\overline{U}_e = R\left\{\overline{I}_1 - \frac{C_a}{C_c}\overline{I}_{c1} + (1 + i\alpha)\frac{ix_H}{R}\left[\overline{I}_1 + \left(1 + \frac{C_H}{C_c}\right)\overline{I}_{c1}\right]\right\}, \qquad (9)$$

where

$$R = \frac{x_a\,x_c}{r_H}. \qquad (10)$$

The results of a full study of the stationary states possible for this self-excited oscillator and of their stability are given in the appendix. We shall restrict ourselves here merely to a consideration of the approximate solution for the stationary state; this solution is adequate in most cases for the design of the self-excited oscillator.

Experience shows that for values encountered in practice, the ratio x_L/R is much less than unity. In this case the resultant de-tuning is a small quantity of the same order of magnitude as x_L/R. It is therefore permissible to neglect in the right half of eq. (9) the terms containing products of the quantities α and x_L/R as being higher-order quantities.

We then obtain instead of (9) the following approximate equation.

$$(1 + i\alpha)\,\overline{U}_e = R\left\{\overline{I}_1 - \frac{C_a}{C_c}\overline{I}_{c1} + \frac{ix_H}{R}\left[\overline{I}_1 + \left(1 + \frac{C_H}{C_c}\right)\overline{I}_{c1}\right]\right\}.$$

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Thus, comparing real and imaginary parts, replacing the capacitances C_a, C_g, and C_q with expressions (1), and carrying out simple transformations, we obtain an equation for the amplitude:

$$I_1 - \frac{C_1}{C_2} I_{c1} = \frac{U_c}{R}$$ (11)

and an expression for the frequency correction

$$\alpha = \frac{x_H}{R} + \frac{x_2}{R} \frac{I_{c1}}{I_1 - \frac{C_1}{C_2} I_{c1}}.$$ (12)

Eq. (12) shows that changing the current ratio I_{g1}/I causes a change in the frequency. The problem of the dependence of the frequency on the operating condition will not be considered further.

Eq. (11) permits determining only the amplitude of the grid voltage; it must be supplemented by equations for the amplitudes of the voltages on the other parts of the internal circuit. This is done most simply by using the value of the feedback coefficient $\overline{K} = \overline{U}_g / \overline{U}_{g2}$. Omitting the computation, we cite expressions for the modulus and phase of the feedback coefficient:

$$K = \frac{C_1}{C_2} \cos \varphi_K,$$ (13)

$$\text{tg } \varphi_K = \frac{x_2}{R} \frac{I_1 + I_{c1}}{I_1 - \frac{C_1}{C_2} I_{c1}}.$$ (14)

The amplitudes of the voltages U_g and U_{g2} are thus related by the expression

$$U_c = U_{c2} \frac{C_1}{C_2} \cos \varphi_\kappa. \tag{15}$$

Computations show that the phase of the feedback coefficient is on the order of 30-40°, corresponding to $\cos \varphi_k = 0.8$. Consequently, the feedback coefficient differs (by approximately 20%) from the capacitance ratio. This is caused by the fact that the branches of the tank circuit in fig. 2 have different impedances because of the active resistance of the quartz crystal, and therefore different currents flow through the branches of the circuit, and the ratio of the voltages across capacitances C_2 and C_1 differs from the ratio of these capacitances.

To compute the amplitude of the voltage across the crystal, it is necessary to determine the modulus of the expression.

$$\bar{U}_{\kappa e} = \bar{U}_c \left(1 + \frac{1}{\kappa} \right). \tag{15a}$$

After transformation, we obtain from this expression:

$$U_{\kappa e} = \left(1 + \frac{C_2}{C_1} \right) \Big/ \sqrt{1 + \frac{\lg^2 \varphi_\kappa}{\left(1 + \frac{C_1}{C_2} \right)^2}} \; U_c. \tag{16}$$

Taking into account the fact that the capacitance ratio C_1/C_2 is usually on the order of 5-7, and that $\mathrm{tan} \varphi_\kappa \leqslant 1$, then, if an insignificant error is allowed, it is possible to replace (16) with a simpler expression:

$$U_{\kappa e} = \left(1 + \frac{C_2}{C_1} \right) U_c. \tag{16'}$$

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Expressions (15) and (16) are useful in experimental study of the oscillator. Voltages U_q and U_{g2} are measured, voltage U_g is computed from (16') and φ_q is determined from (15).

3. COMPUTATION OF OPERATING CONDITIONS OF SELF-EXCITED OSCILLATOR AND EXPERIMENTAL VERIFICATION

To compute the stationary state we replace the static characteristics of the emission and grid currents with segments of straight lines, as shown in fig. 4. The same figure shows the plot of the alternating grid voltage and shows the cutoff angles, as determined from the following equations:
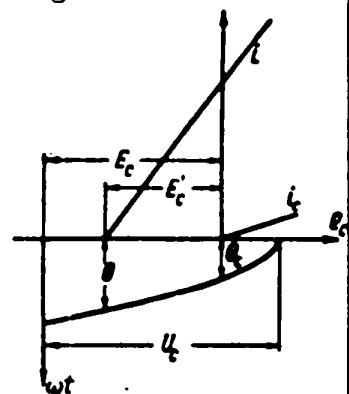
$$\cos\Theta = -\frac{E_c - E_c'}{U_c}, \qquad (17)$$

$$\cos\Theta_c = -\frac{E_c}{U_c}. \qquad (18)$$

Here E_g is the bias, and the significance of E_g' is clear from fig. 4. The harmonics of the currents are determined in the following manner:

$$I_n = S\,U_c\,\gamma_n(\Theta), \qquad (19)$$

$$I_{cn} = S_c\,U_c\,\gamma_n(\Theta_c). \qquad (20)$$

Puc. 4

Tables for the functions $\gamma_n(\Theta)$ are found in reference[1].

The computation of the stationary state reduces to the following:

Eliminating E_g from eq. (17) and (18) we obtain:

$$U_c = \frac{E_c'}{\cos\Theta_c - \cos\Theta}. \qquad (21)$$

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To determine the cut-off angles θ and θ_g it is necessary to make use of the equations that relate the currents and the voltages in the stationary state. For the fundamental frequencies this relationship is given by eq. (11) obtained above.

The relationship for the d-c components is obtained by returning to the circuit of fig. 1, which shows that the bias voltage is produced by the voltage drop across R_g due to the d-c component of the grid current and by the voltage drop across R_a due to the d-c component of the emission current

$$ -E_c = I_{c0}R_c + I_0 R_a. \tag{22} $$

Substituting now (18), (19), and (20) into (11) and (12), and making certain rearrangements, we obtain equations for computing the cutoff angles:

Puc. 5
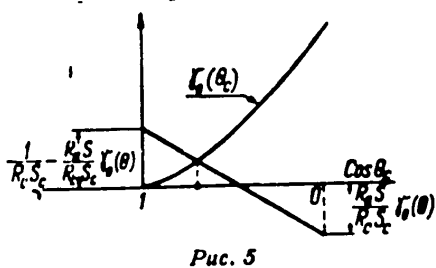
$$ \gamma_1(\theta) = \frac{1}{RS} + \frac{C_1}{C_2}\frac{S_c}{S}\gamma_1(\theta_c). \tag{23} $$

$$ \gamma_0(\theta_c) = \frac{\cos\theta_c}{R_c S_c} - \frac{R_a S}{R_c S_c}\gamma_0(\theta). \tag{24} $$

Inasmuch as the second term of the right half of (23) is small, it is possible to employ the method of successive approximations. The first step in the solution is to ignore this term, and write (23) in the following form:

$$ \gamma_1(\theta) = \frac{1}{RS}. \tag{24a} $$

Next, using the approximate value of θ, the value of θ_g is determined by graphically solving (24). This method of solution is explained in Fig. 5.

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Using the value of θ_g thus obtained, it is necessary to find another, more

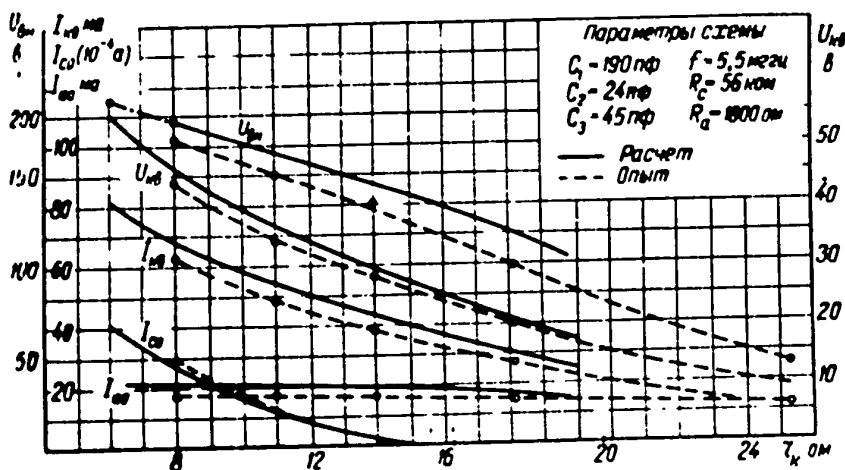accurate value of θ using equation (23), but without omitting the second

Figure 6 - Circuit Parameters; C_1 = 190 picofarad f = 5.5 mc;
C_2 = 24 picofarad R_g = 56000 ohm; C_3 = 45 picofarad
R_a = 1800 ohm; _____ computed; _ _ _ _ experimental

term of the right half. Next, if necessary, it is possible to repeat the

graphic solution of eq. (24) to find a more accurate value of θ_g.

If $R_a S \gamma_0 (\theta) > 1$, the graphic solution of eq. (24) can be omitted.

In this case $\gamma (\theta_g) = 0$ and $\cos \theta_g = R_a S \gamma_0 (\theta) > 1$. This means

that $E_b > U_g$, i.e., the circuit operates without grid current. The resultant

value $\cos \theta_g > 1$ must be used in equation (21), which in this case remains valid.

In conclusion, let us see what extent the above computations are corro-

borated experimentally.

The curves of Fig. 6 show the dependence of the operation of the self-

excited oscillator on the active resistance of the quartz crystal. Shown

here are the d-c components of the anode and grid currents I_{a0} and I_{g0}, the

amplitude of the crystal voltage U_q, the current in the crystal I_q, and the

amplitude of the voltage in the external circuit U_{ext}, tuned to the second

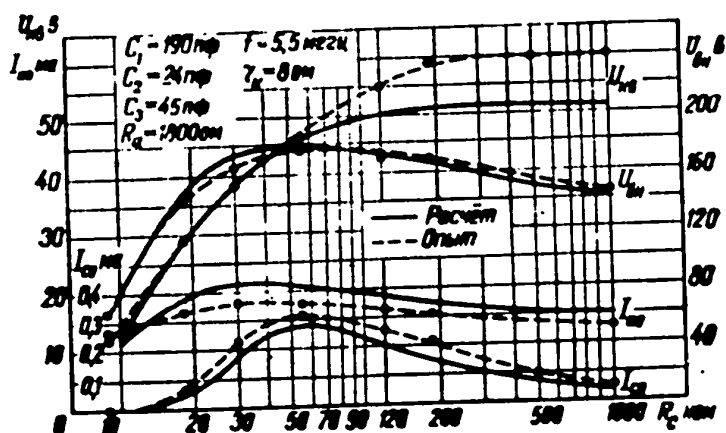harmonic of the anode current. As can be seen, all computed curves agree

Figure 7 - C_1 = 190 picofarad; C_2 = 24 pf.; C_3 = 45 pf.;
R_a = 1800 kilohm; 7 = 8 kilohm; _____ computed;
_ _ _ experimental.

with the experimental ones. The curves were plotted as follows: A resistor
was connected in parallel with the crystal, and the shunting action of the
resistor was taken to be an equivalent change in the activity resistance of
the crystal. To prevent the bias resistance from changing, a blocking cap-
acitance was connected in series with the shunting resistor.

Fig.   shows a series of computed and experimental curves, showing the
dependence of the operating state of the self-excited oscillation on the change
in the self-bias resistance R_g. For a certain value of R_g, the amplitude of the
voltage in the external circuit will have a maximum. This is caused by the fact
that, as R_g increases, two phenomena take place: on one hand, there is a decrease
in the angle Θ_g, and on the other, there is a decrease in angle Θ because of the
reduced shunting effect produced at the shunt by R_g. The curves of fig. 7 show
that the computations agree satisfactorily with the experiments.

APPENDIX
(Beginning of finer print in original-Ed.)
The appendix gives the result of a fuller study of the phenomena in the
self-excited oscillator, and devotes some attention to the phenomenon of
oscillatory hysteresis.

The stationary-state equation obtained from the complete equation (9)
can be written, after a few transformations, in the following form:

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For the amplitude of the self-excited oscillations:

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$$I_1 - \frac{C_1}{C_2} I_{c1} - \frac{U_c}{R_y} , \qquad\qquad (I)$$

where the control resistance is

$$R_y - \frac{R}{1 + a^2} ; \qquad\qquad (II)$$

for the frequency correation:

$$\frac{a}{1 + a^2} - \frac{x_\mu}{R} + \frac{x_2}{R} \frac{I_{c1}}{I_1 - \frac{C_1}{C_2} I_{c1}} . \qquad\qquad (III)$$

Equation (13) remains valid for the modulus of the feedback coefficient, while its phase is determined from the expression:

$$\text{tg } \varphi_\kappa - \frac{x_2}{R_y} \frac{I_1 + I_{c1}}{I_1 - \frac{C_1}{C_2} I_{c1}} \qquad\qquad (IV)$$

Comparison of these equations with (11), (12), and (14) obtained above shows that their simplification is equivalent to eliminating from the exact equations the term $(1 + \alpha^2)$. This results not merely in a quantitative error, but also in a qualitative one. The approximate solution to the frequency equation becomes single-valued instead of double-valued, and the oscillatory-hysteresis phenomenon is lost from the result.

Let us study the equations obtained. According to eq. (1), amplitude of the grid voltage U_g depends on the value of the control resistance R_c and is proportional to it under certain conditions. Consider the expression for R_c; at the same time we shall neglect the grid current in the frequency equation. We then obtain the following set of equations:

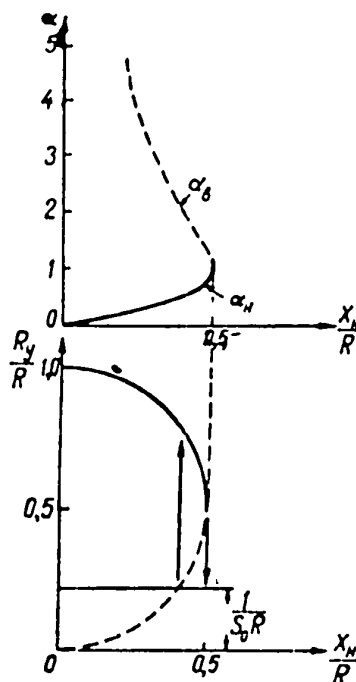$$R_y - \frac{R}{1 + a^2} , \qquad\qquad (II)$$

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Figure 8
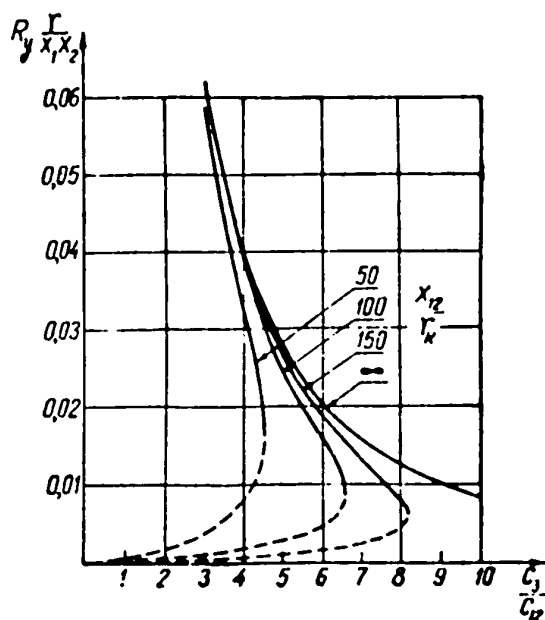

Figure 9

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$$\frac{a}{1 + a^2} = \frac{x_H}{R}. \tag{III}$$

The curves for the dependence of the frequency correction and of the control resistance on the reactance of the cathode circuit, plotted from above equations, are given in fig. 8.

If $x_L < 0.5R$ and x_L is prescribed, we obtain two values of frequency, two values of control resistance, and hence two stationary states. Fig. 8 shows two branches of the curves –– solid and dotted. The frequency corresponding to low values of α will be called the lower frequency and denoted α_L. The frequency corresponding to the higher values of α we shall call the upper frequency and denote α_U. It is evident from fig. 8 that if the reactance x_L is so high that $x_L > 0.5R$, no self excitation is possible.

Thus, we obtain here the same phenomenon with double-valuedness of the frequency and control resistance that was described earlier [2,3] in connection with other self-excited crystal-oscillator circuits. The further discussion will therefore be brief.

The investigation performed (which will not be reported here) showed that under certain assumptions, the stability conditions of the stationary states assume the form

$$R_{yм} I_1' < 1,$$
$$R_{yе} I_1' > 1. \tag{IIIa}$$

Here the prime denotes the derivative of the current I_1 with respect to the amplitude of U_g at the stationary-state point. For the lower frequency, the stability condition coincides with the well-known condition for the

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conventional self-excited oscillator. For the upper frequency, the stable states are those unstable at the lower frequency, and vice versa. If a "soft" state is considered, i.e., one in which the average slope I_1/U_g drops monotonically as U_g increases, the lower frequency is always stable, and the upper one is unstable.

It is possible to obtain from the stability conditions the conditions under which the oscillator becomes self-excited; this condition reduces to satisfying the following two inequalities:

$$R_{yx} S_0 > 1, \qquad R_{ys} S_0 < 1, \qquad \text{(IIIb)}$$

where S_0 is the slope of the emission current at the equilibrium point.

The results obtained allow us to deduce that under certain conditions it is possible to observe in the above self-excited-oscillator system the phenomenon of oscillatory hysteresis, consisting of the following.

Let the slope at the equilibrium point be such that the inequality $S_0 R > 2$ is satisfied. Then the line corresponding to S_0 will occupy on the plot of $R_c(x_L)$ the position shown in fig. 8.

Let us see how the amplitude of the self-excited oscillation will vary with x_I. It is evident that wherever the S_0 line is above the dotted part of the $R_c(x_L)$ line, the condition for self excitation is satisfied, and where the S_0 line goes below both $R_c(x_L)$ lines (solid and dotted), the condition for self-excited oscillation is not satisfied. Under "soft" operating conditions the stationary state for the lower frequency is stable, and as x_I changes, U_g will vary approximately proportionally to R_{cL}. It is evident that, as x_I increases enough to equal $0.5R$, the self oscillations will stop at a finite amplitude. If x_I is now reduced, the oscillations will resume only after x_I reaches a value at which the self-excitation condition

is again satisfied. The points where the oscillations stop and are resumed are indicated with arrows on fig. 8.

The above phenomena can be obtained by introducing a variable capacitance into the cathode circuit of the circuit of fig. 2, forming thereby a circuit similar to the equivalent circuit of fig. 3. However, the same phenomena will occur also in the circuit of fig. 2, if any one of the capacitances C_1, C_2, or C_3 is varied. As an example, consider the variation of $R_c(C_3)$. According to (1), whenever C_3 is varied, C_k varies in almost inverse proportion, and C_a and C_g also vary. The proper computations yielded the curves shown in fig. 9, in which the following symbols are introduced:

$$x_1 = \frac{1}{\omega C_1} ; \quad x_2 = \frac{1}{\omega C_2} ; \quad x_{12} = \frac{1}{\omega C_{12}} ; \quad C_{12} = \frac{C_1 C_2}{C_1 + C_2}. \tag{IIIC}$$

The parameter of the family of curves of fig. 9 is the quantity x_{12}/r_k. When x_{12}/r_k is infinite, we obtain the same expressions as result from the approximation $R_c = R$. Comparison of the curve for infinite x_{12}/r_k with the remaining curves shows the extent of error due to the approximate equation.

The curves of Fig. 9 show that varying capacitance C_3 may result in the phenomenon of oscillatory hysteresis, as described with the aid of fig. 8.

The equations obtained indicate at which values of the parameters it is possible to disregard the double-valuedness of the frequency and use the approximate equations. Inasmuch as the latter are obtained by assuming α^2 to be negligible as compared with unity, it is possible to assume that the approximate equations are valid if $\alpha^2 = 0.1$. The same figure contains a curve that corresponds to the values of the capacitance, at which the

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The phenomenon of oscillatory hysteresis is of both a theoretical and practical interest. The occurrence of this phenomenon in self-excited crystal oscillators was first described in references [2]. Such phenomena, as described above, have another feature; they are associated either with "stiff" operating conditions, or else with coupling-hysteresis effects in systems having two degrees of freedom. It must be noted that although references [2,3] contained an analysis of tuned circuits, the problem there was simplified and reduced to that for a system with one degree of freedom. This simplification was caused by the great difference in damping between the quartz and the tuned circuit, so that the latter could be considered as a complex impedance. As far as the circuit discussed here is concerned, the theory developed above changes little if the crystal is replaced by a simple tuned circuit.

In recent years the phenomenon of oscillatory hysteresis has been applied to measurements of the ohmic resistance of quartz crystals [4]. The principle of the measurement reduces to the determination of the parameters of the circuit corresponding to the point at which oscillations stop because of the hysteresis; the resistance can then be determined, for example, using the graph of fig. 10, or computed from the corresponding equation. The result of the measurement is independent of the tube characteristics.

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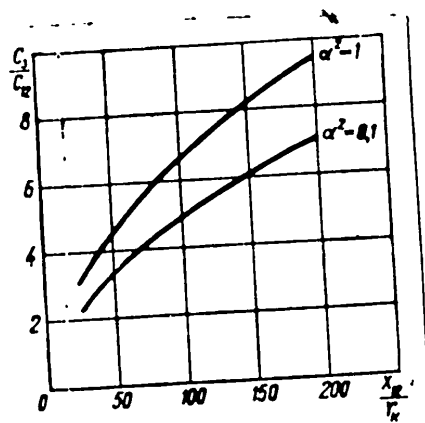stal-oscillator Circuits. PIEE, p. III, No. 62, November 1952.

Figure 10

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STABILITY OF QUASI-HARMONIC VACUUM-TUBE OSCILLATORS WITH THERMISTORS

L. I. Kaptsov.

The article contains an experimontally-verified theoretical analysis of the frequency and amplitude stability of two types of quasi-harmonic vacuum-tube oscillators employing non-linear inertia-type amplitude re ulators in the form of thermistors, or heat-sensitive resistors.

1. INTRODUCTION

The thermistor is used most frequently in quasi-harmonic vacuum-tube oscillators as a non-linear inertia type regulator for the amplitude of the self-excited oscillations. The thermistor can also act here as a non-linear amplitude limiter, thereby permitting the use of only the nearly linear portion of the characteristic of the oscillator tube. The latter insures a low harmonic content in the generated oscillations (1 to 4). Amplitude stabilization with a thermistor insures the minimum change of the oscillation waveform with time, i.e., a reduction in the change of the harmonic content of the oscillations with time. All this is well known to increase the frequency stability of the generated oscillations.

On the other hand, the resistance of the thermistor affects the frequency of the generated oscillations to a certain extent. Consequently the change in the thermistor resistance, which always takes place when the thermistor regulates the amplitude, causes a change in the frequency of the generated oscillations and lowers the frequency stability of the oscillator.

Thus the possibility of increasing the frequency stability of the oscillator with the aid of a thermistor for amplitude stabilization requires further study.

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The great variety of oscillator circuits and of methods of connecting the thermistors makes it difficult to establish general rules for computing the stability of these oscillators. However, it is possible theoretically to determine individually, for each type of oscillator, the most suitable conditions for connecting the thermistors. That there is a practical need for such a theory goes without saying.

Given below is an experimentally-verified theoretical analysis of the stability of two types of quasi-harmonic vacuum-tube oscillators employing thermistors.

2.   THE CIRCUIT OF COMPUTING STABILITY OF FEEDBACK OSCILLATORS

The principal diagram of the investigated oscillator is given in Fig. 1. The oscillator contains a bridge circuit for amplitude stabilization, consisting of four resistances, and one of these, r_3, is replaced with a thermistor having a negative temperature-resistance coefficient. The frequency and amplitude stabilities were measured experimentally with the oscillator operating with and without the thermistor.

The oscillator equation, written as a function of the current in its tuned circuit, has the form:

$$I + 2\delta \left(\frac{M_1 M_2 S}{\beta R_1 C_1 L_4} - 1 \right) \dot{I} + \omega_0^2 I = 0, \tag{1}$$

where $\omega_0 = \dfrac{1}{\sqrt{L_0 C_0}}$ — resonant frequency of the tuned circuit

$2\delta = \dfrac{R_1}{2L_1}$ — damping coefficient of the tuned circuit

M_1 and M_2 — mutual inductance coefficients in the feedback circuit

S — mutual conductance of oscillator tube

β — attenuation coefficient of bridge, determined as the ratio

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of the volt'ge at the input in onal ab of the bridge to the

voltage at its output ' on-l cd. All other symbols are clear

from Fi . 1.

During the settling-down time of the steady-state amplitude of the

self-excited oscillations of a thermistor-equipped oscillator, the coeffi-

cient of the dissipation term of eq. 1, which will be denoted from now on

as $\varphi(I_o)$, varies because the attenuation coefficient of the bridge depends

on the amplitude I_o of the oscillations. At the same time, the mutual con-

ductance of the tube characteristic remains almost constant for small amplitudes.

The stationary amplitude I_o^{st} satisfies the following equation:

$$\varphi(I_0^{cm}) = 2\delta\left(\frac{M_1 M_2 S}{\beta(I_0^{cm}) R_1 C_1 L_4} - 1\right) = 0; \qquad (2)$$

consequently

$$\beta^{cm} = \beta(I_0^{cm}) = \frac{M_1 M_2 S}{R_1 C_1 L_4}. \qquad (3)$$

Stationary self-excited oscillations that are stable in amplitude take

place when the derivative $\frac{d\varphi}{dI_o}$ is negative at the point I_o^{st}, and this is the

reason for choosing a negative temperature-resistance coefficient for the

thermistor.

We restrict ourselves to only short-time changes in the operating con-

ditions of the system in the vicinity of the mean stationary state, and dis-

regard any monotonic changes of this state (such restriction is in full agree-

ment with conditions prevailing if the equipment is already warmed-up, is fed

from a stabilized supply, and is not subject to large temperature changes),

then the amplitude stability of the oscillations depends on the absolute

value of the derivative $\left.\dfrac{d\varphi}{dI_o}\right|I_o = I_o{}^{st}.$

Computation yields

$$\left.\frac{d\varphi}{dI_0}\right|_{I_0 = I_0^{cm}} = -\frac{M_1 M_2 S}{R_1 C_1 L_4} \cdot \left.\frac{\partial}{\partial I_0}\frac{1}{\beta I_0}\right|_{I_0 = I_0^{cm}} . \qquad (4)$$

Determining β (I_o) from the bridge parameters under the assumption

that the resistances loading the bridge diagonal are large compared with

the input and output resistances of the bridge, we get:

$$\beta(I_0) = \frac{[r_4 + r_2(I_0)]\cdot(r_1 + r_3)}{r_2(I_0)\cdot r_3 - r_1\cdot r_4} . \qquad (5)$$

Taking eq. (3) into account, we finally get

$$\left.\frac{d\varphi}{dI_0}\right|_{I_0 = I_0^{cm}} = \beta(I_0^{cm})\cdot \frac{r_4}{(r_4 + r_2)^2} \cdot \left.\frac{\partial r_2}{\partial I_0}\right|_{I_0 = I_0^{cm}} . \qquad (6)$$

It follows from (6) that for the stationary oscillations to be stable,

β $(I_o{}^{st})$ and the derivative $\left.\dfrac{dr_2}{dI_o}\right|I_o = I_o{}^{st}$ should be of opposite signs.

It follows also from eq. (4) and (6) that the amplitude stability of

the oscillations increases with the coupling between the bridge and the

oscillator, with the mutual conductance of the tube characteristic, with

the slope of the thermistor at the operating point (with the partial deriva-

tive $\left.\dfrac{\partial r_2}{\partial I_o}\right|I_o = I_\bullet^{st}$ and decreases with the resistance of

the tuned oscillator circuit.

The bridge resistance r_{cd} along the diagonal cd (Fig. 1) loads the

tuned circuit of the oscillator directly through coupling coil L_4.

Neglecting the parasitic parameters of the circuit and the plate reaction

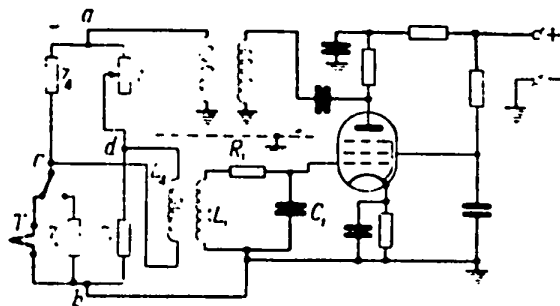of the tube, the oscillating frequency can be assumed to   equal the

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Figure 1

resonant frequency f_0 of the system shown in Fig. 2. This frequency can be determined from the equation:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{L_4}{C_1(L_1 L_4 - M_1^2)} + \frac{r_{cd} \cdot H_1}{(L_1 L_4 - M_1^2)}} \quad . \tag{7}$$

or by using eq. (3)

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{L_4}{C_1(L_1 L_4 - M_1^2)} + \frac{r_{cd} M_1 M_3 S}{\beta^{cm} C_1^2 L_4(L_1 L_4 - M_1^2)}} \quad . \tag{8}$$

To compute from eqs. (7) and (8) the change in oscillation frequency due to a change in the resistance of the thermistor as the latter regulates the amplitude, we shall use experimental data on the measurement of the amplitude stability of the oscillator when the latter operates with and without the thermistor.

The relative instability s_a of the oscillation amplitude is defined as the ratio $\frac{\Delta I_0}{I_0^{st}}$, where ΔI_0 is the absolute value of the amplitude deviation from its stationary value I_0^{st}, averaged over a time interval of sufficient practical size; this instability turned out to equal:

(1) $s_{a_1} = 1.67 \times 10^{-1}$ for an oscillator operating without a thermistor.

(2) $s_{a_2} = 3.76 \times 10^{-3}$ for an oscillator operating with a thermistor.

Thus the use of the thermistor lowers the relative amplitude instability of the oscillator by an amount $\Delta s_a = s_{a_1} - s_{a_2} = 1.63 \times 10^{-1}$. It is easy to show that the accompanying change $\Delta \beta$ in the bridge attenuation coefficient is related to Δs_a by the equation:

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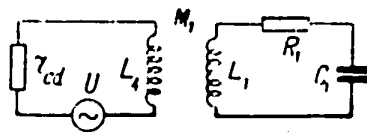
Рис. 2

Figure 2

$$\Delta \beta = \beta^{cm} \Delta s_a. \tag{9}$$

Using this equation and expression (5), we find:

$$\Delta r_2 = \frac{\Delta s_a (r_1 + r_2)^2}{\beta^{cm} r_1}. \tag{10}$$

Knowing the value of Δr_2, it is easy to compute the corresponding value of the change Δr_{cd} in the bridge resistance along the diagonal cd, and the resultant oscillation-frequency deviation Δf produced in the stationary frequency f^{st}, as follows from equation (7).

Computations show that in the case of the oscillator investigated, Δf equals 62 cycles when $\Delta s_a = 1.63 \times 10^{-1}$ and the oscillation frequency is $f^{st} = 1.2 \times 10^5$ cycles. Thus the computed value of the lower threashold of the relative oscillation-frequency instability, s_f min (determined in the same manner as the relative instability of the oscillation amplitude), in the case of the thermistor-equipped oscillator, will equal:

$$s_{f_{MUH}} = \frac{\Delta f}{f^{cm}} = 5.2 \cdot 10^{-5} . \tag{10a}$$

The resultant experimental values of the relative frequency instability of the investigated oscillator are:

$$s_{f_1} = 2.0 \times 10^{-5}; \quad s_{f_2} = 4.6 \times 10^{-5}. \quad \text{(During oscillator operation without a with thermistor, respectively).}$$

These data are in close enough agreement with the results of the computations, and this permits drawing the following conclusions:

1. If the circuit parameters are properly chosen, the absolute value of the increase in the frequency stability of the oscillation, due to the oscillation-waveform stabilization produced by the thermistor, is

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less than the reduction in the frequency stability due to the change in the thermistor resistance during the amplitude-regulation process.

2. It follows from equation (10) that increasing β^{st} and keeping the coupling between the bridge and the amplifier constant increases the frequency stability of the oscillator by reducing Δr_{cd}. In addition, it follows from equation (6) that increasing β^{st} permits the use of a less sensitive thermistor for a prescribed value of amplitude stabilization, and this reduces Δr_{cd} still further.

An increase in β^{st} is equivalent to a decrease in the feedback coefficient and calls for an increase in the amplification factor of the amplifying portion of the circuit if oscillation is to be maintained.

3. According to equation (8), the frequency stability of the oscillator can be increased by reducing the coupling between the bridge and the amplifying portion of the system, maintaining β^{st} and consequently also the amplitude stability of the oscillations constant, as follows from equation (4). In accordance with equation (3), the condition that β^{st} be maintained constant necessitates the fulfillment of at least one of the following conditions:

(a) increase the mutual-conductance of the oscillating-tube,

(b) decrease the active resistance of the oscillator tuned circuit.

Thus, according to paragraphs 2 and 3, for a prescribed value of the amplitude stability of the oscillations the frequency stability of an oscillator using a bridge-type feedback circuit with a thermistor is

directly proportional to the mutual conductance of the oscillating tube and to the Q of its tank circuit.

4. The use of a low-sensitivity thermistor and a high-Q quartz crystal in an analogous type oscillator constructed by Meacham [4] insured high frequency stability in this oscillator.

3. USE OF THERMISTOR IN AN OSCILLATOR CONTAINING A CATHODE-COUPLED AMPLIFIER

The problem of introducing an inertia-type non-linear amplitude regulator of the thermistor type into an oscillator containing a cathode-coupled amplifier was investigated theoretically by G. A. Khavkin [5]. However, he merely computed the amplitude stability of the oscillator. Given below is a computation of the frequency stability, corroborated by experimental measurements of the frequency and amplitude stability of this type of oscillator employing a thermistor.

The circuit of the investigated oscillator is given in Fig. 3.

To evaluate the effect of changes in thermistor resistance r_T on the oscillation frequency, it is necessary to derive an equation for the oscillator in which the anode reaction of tube V_2 is taken into account. This equation, written in terms of the current I in the capacitor branch of the tank circuit, has the following form:

$$\ddot{I} + \left(\frac{R}{L} + \frac{1}{C}A\right)\dot{I} + \frac{1}{LC}(1 + AR)I = 0, \qquad (11)$$

where

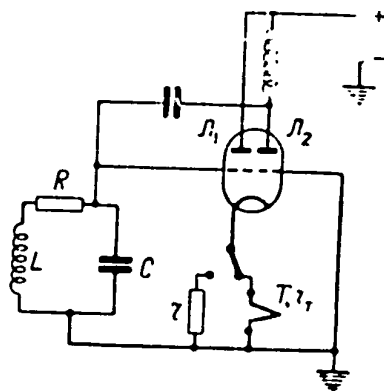$$A = \frac{1}{R_{12}} - \frac{\mu_1 S_2 \cdot_T}{R_{12} + r_T(1 + \mu_1)}, \qquad (12)$$

Figure 3

R_{i1} and R_{i2} are the internal resistances of tubes V_1 and V_2, respectively.

μ_1 - amplification factor of tube V_1;

S_2 - mutual conductance of tube V_2;

the remaining symbols are clear from Fig. 3.

According to equation (11) the oscillation frequency is determined from the following equation:

$$ f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}\left[1 + \frac{R}{R_{i2}} - \frac{R\,\mu_1\,S_2\,r_T}{R_{i1} + r_T\,(1+\mu_1)}\right]} . \qquad (13)$$

The range of variation Δr_T of the thermistor resistance as the thermistor performs the amplitude stabilization is found by measuring experimentally the dependence of the oscillation amplitude on the cathode-coupling resistance and by knowing experimental data on the change in the relative amplitude instability of the oscillator as the thermistor is switched in. For the oscillator under investigation, the following results were obtained:

Oscillator operating without thermistor - $s_{a1} = 4.4 \times 10^{-3}$;

Oscillator operating with thermistor - $s_{a2} = 1.0 \times 10^{-4}$.

Using these data for the investigated oscillator operating at an oscillation frequency of 800 kc., the resultant value of $s_{f\,min}$ was 6.25×10^{-6}.

Experimental determination of the relative frequency instability of the oscillator under investigation showed it to be:

(1) oscillator operating without thermistor - $s_{f1} = 4.6 \times 10^{-5}$;

(2) oscillator operating with thermistor - $s_{f2} = 1.9 \times 10^{-5}$.

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These results do not contradict the computed value of $s_{f\ min}$ and show that under the given operating conditions of the oscillator, the use of the thermistor increases both the amplitude and frequency stability.

According to equation (13) it is possible in oscillators of the type investigated to reduce the value of $s_{f\ min}$, for a prescribed value of amplitude stability, by decreasing the resistance of the tank circuit. It is useless to decrease the mutual transconductance S_2 of the second tube of the amplifier, for in accordance with reference[5] the amplitude stability of the oscillations is directly proportional to S_2.

In conclusion, the author expresses his deep gratitude to Professor K. F. Teodorchik for suggesting the topic and for constant attention to this work.

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ANALYSIS AND COMPUTATION OF WIDE-BAND PHASE-SHIFTING CIRCUIT

B. B. Shteyn

The article contains an analysis and computing procedure for an R-C wide-band phase-shifting circuit[2]. The dependence of the constancy of the phase shift between the output voltages on the shift angle and on the prescribed frequency band is shown. The computation method developed here permits determining in advance the degree of constancy of the phase shift when the shift angle and the frequency range are prescribed, or else solving the inverse problem. Equations are given with which to determine the basic circuit data.

1. INTRODUCTION

In recent years it has become possible to use two-phase[1, 2] or three-phase[3] modulation systems, in addition to the widely known balanced system, to produce single-band telephony. The two- and three-phase systems require that the phases of high- and low-frequency voltages be shifted by definite angles. The low-frequency phase shift should be accomplishable over a wide frequency band, and this leads to considerable difficulties. The accuracy of the phase shift must be very high, since on it depends the effectiveness of suppression of the unnecessary sideband.

Another requirement on a wide-band phase-shift circuit is that the output voltage must be frequency independent. Analysis shows that in a three-phase modulation system it is possible to insure suppression of the second sideband by 40 db with a phase deviation of the low-frequency voltage not more than $\pm 1°$ over a wide range.

2. THEORETICAL PART

A constant, frequency-independent phase shift between two voltages can be obtained when the phase of each voltage varies logarithmically[2]. Thus, if voltages of equal phase are applied to the inputs of two four-terminal networks, the output voltages differ in phase by:

$$\varphi_1 - \varphi_2 = C_1 - C_2 + \ln \frac{K_1}{K_2} = \text{const}, \qquad (a)$$

where $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (b)

$$\varphi_1 = C_1 + \ln K_1 F; \quad \varphi_2 = C_2 + \ln K_2 F.$$

This work will consider the very case of obtaining a frequency-independent phase shift, and a solution is hardly of interest in engineering problems if the actual circuit insures the production of a phase with a very-nearly logarithmic variation.

It is possible to show that the circuit of an active four-terminal network, shown in Fig. 1, satisfies this requirement and at the same time insures a constant voltage-transformation coefficient, independent of the frequency[3]. Here the potential on the grid phase as the potential on the control grid of the tube. It is therefore enough to determine the phase shift between the output voltage and -e. Neglecting as a first approximation the shunting effect of resistors R_c and R_b, we can restrict ourselves to the equivalent circuit of Fig. 2.

a. Certain basic equations:

$$e = I_1 Z_1 + I_3 Z_3, \quad e = I_2 Z_2 - I_3 Z_3, \quad c_0 = I_3 Z_3, \qquad (c)$$
$$I_1 - I_2 - I_3 = 0 \text{ и } I_3 = I_1 - I_2,$$

we obtain after a few transformations:

$$c_0 = -e \frac{\dfrac{Z_2}{Z_2} - \dfrac{Z_3}{Z_1}}{1 + \dfrac{Z_2}{Z_1} + \dfrac{Z_3}{Z_2}}. \qquad (1)$$

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The significance of Z_1, Z_2, and Z_3 is clear from Fig. 2.

When the relationships, given below, between C_2 and C_1, R_2 and R_1, etc. are satisfied, equation (1) can be rewritten as:

$$e_0 = - e(1 - 4a)\frac{(F^2 - F_c^2) + iSFF_0}{(F^2 - F_0^2) - iSFF_0},\qquad(1')$$

where

$$a = \frac{C_2}{C_1};\quad R_2 = \frac{R_1}{a};\quad R_3 = \frac{1-4a}{4a}R_2;\quad C_3 = \frac{4a}{1-4a}C_2;$$

$$S = \frac{1-2a}{a}\quad \text{и}\quad F_0 = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi R_3 C_3};$$

and F_0 is the parametric frequency of the four-terminal network. (Equations (1) and (1') were derived by G. G. Varganov and used by him for the analysis of a similar four-terminal network.)

It follows from (1') that the voltage-transformation coefficient of the given four-terminal network equals $1-4a$, is constant, and is independent of the frequency.

The phase of the output voltage can be determined from (1'):

$$tg\frac{\varphi}{2} = \frac{SFF_0}{F^2 - F_0^2} = \frac{S}{\frac{F}{F_0} - \frac{F_0}{F}} = \frac{S}{x - \frac{1}{x}},\qquad(2)$$

where

$$x = \frac{F}{F_0}.\qquad(2a)$$

Equation (2) shows that as x approaches zero, i.e., F approaches zero, $\tan\frac{\varphi}{2}$ tends toward zero and remains negative. It can be shown that at the same time $\cos\varphi$ remains positive. Consequently, as F varies from zero to any positive value, φ will acquire negative values. This fact, and also the fact that $\tan\varphi/2$, according to equation (2), is positive for angles between
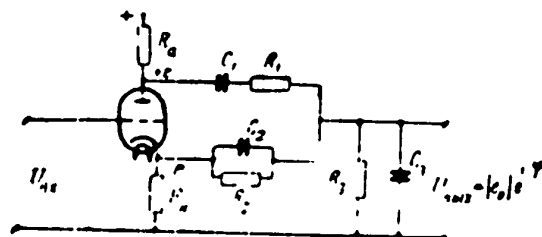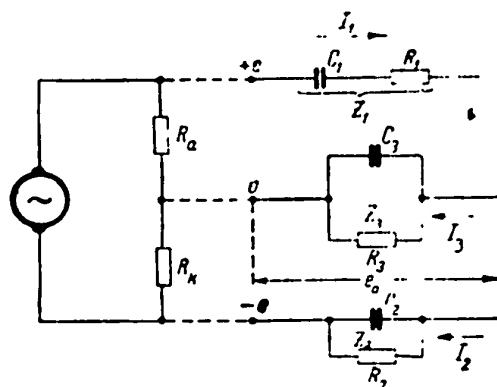
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Figure 1

Figure 2

$-\pi/2$ and $-\pi$, must be taken into account when performing calculations with this equation.

Thus, the final expression for the output voltage will assume the form:

$$e_i = -e(1-4a)e^{i2\operatorname{arc}\ tg\frac{S}{x-\frac{1}{x}}}. \quad (3)$$

The frequency dependence of the phase, determined by equation (2), does really approach logarithmic behavior (within certain limits). To prove this, we introduce the substitutions:

$$x = e^{\ln x} \quad \text{и} \quad \frac{1}{x} = e^{-\ln x}. \quad (2a)$$

Equation (2) can then be rewritten as:

$$tg\frac{\varphi}{2} = \frac{S}{2\ sh\,(\ln x)}, \quad (2')$$

or   ....

$$\frac{\varphi}{2} = \operatorname{arc}\ tg\frac{S}{2sh\,(\ln x)}. \quad (2'')$$

Equation (2) can be expanded into a series for the values:

$$\frac{S}{2\,h\,(\ln x)} = \frac{S}{x - \frac{1}{x}} < -1,$$

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i..., for

$$x > -\frac{S}{2} + \sqrt{\frac{S^2}{4} + 1}$$ (2"a)

Then

$$\frac{\varphi}{2} = \arctan\frac{S}{2\mathrm{sh}(\ln x)} = -\left[\frac{\pi}{2} + \frac{2\mathrm{sh}(\ln x)}{S} - \cdots\right].$$ (2"b)

Furthermore, using the series

$$\mathrm{sh}(\ln x) = \ln x + \frac{(\ln x)^3}{3!} + \cdots$$ (2"c)

and restricting ourselves to the first term, we get:

$$\frac{\varphi}{2} \approx -\left(\frac{\pi}{2} + \frac{2}{S}\ln x\right).$$ (4)

When x = 1, $\varphi/2 = -\pi/2$. This also follows from eq. (2) and (4).

For x > 1, (and if) condition $\dfrac{S}{x - \dfrac{1}{x}} > 1$ is satisfied, meaning that

$$x < \frac{S}{2} + \sqrt{\frac{S^2}{4} + 1}$$ is satisfied, expression (2") can be expanded into

series $\dfrac{\varphi}{2} = \dfrac{\pi}{2} - \dfrac{\sinh(2\ln x)}{2} + \quad \cdots$

However, considering that when $x > 1$, $\varphi/2$ becomes negative in the fourth quadrant, i.e., between $\mathcal{T}/2$ and $-\mathcal{T}$, we should rewrite the last series as:

$$\frac{\varphi}{2} = -\frac{\pi}{2} - \frac{2\text{sh}(\ln x)}{S} + \cdots = -\left[\frac{\pi}{2} + \frac{2\text{sh}(\ln x)}{S} - \cdots\right]. \qquad (4a)$$

Consequently, for values of x within the above-mentioned limits, i. e.,

$$-\frac{S}{2} + \sqrt{\frac{S^2}{4} + 1} < x < + \frac{S}{2} + \sqrt{\frac{S^2}{4} + 1}, \qquad (4b)$$

the actual expansion in series is in the form:

$$-\frac{\varphi}{2} = -\left(\frac{\pi}{2} + \frac{2}{S}\ln x - \cdots\right)$$

$$(4c)$$

or

$$\varphi \approx -\left(\pi + \frac{4}{S}\ln x\right). \qquad (4')$$

Thus, the dependence of the phase angle, as given by eq. (2), does not have a strictly logarithmic variation. It is necessary to establish the limits within which this dependence is in close agreement with the logarithmic one.

If the phase angle is computed from eq. (3), the error as compared with the results obtained from the exact equations (2') is -5% and -3% for x = 0.3 and x = 8 respectively, when S = 4. (The choice of the value of S will be considered below). Consequently, the phase of the four-terminal network under consideration, as determined from eq. (2'), has a nearly logarithmic variation. Nevertheless, the above deviation from logarithmic behavior is proof of the applicability of the phase-shift network of fig. 1 within a limited frequency band.

As has already been shown, the required constant phase shift in the frequency range can be obtained by means of the two four-terminal networks of fig. 1. Let us denote the phase difference of the two four-terminal networks by $\theta = \varphi_1 - \varphi_2$. Assume that to some average frequency $F = F_{av}$ there corresponds:

$$\varphi_1 = - \left(\pi - \frac{\theta}{2}\right), \quad \varphi_2 = - \left(\pi + \frac{\theta}{2}\right) \text{ and } \varphi_1 - \varphi_2 = \theta. \qquad (4'a)$$

It follows from the logarithmic frequency dependence that the following condition must be satisfied:

$$\ln F_{cp} = \frac{1}{2}(\ln F_{\text{мин}} + \ln F_{\text{макс}}), \qquad (4'b)$$

hence

$$F_{cp} = \sqrt{F_{\text{макс}} \cdot F_{\text{мин}}}, \qquad (4'c)$$

where F_{min} and F_{max} are the minimum and maximum frequencies, respectively, of the prescribed band.

Turning now to determination of the basic parameters of the phase-shift four-terminal networks, it is necessary to use the actual equations, i.e., equations (2) and (2').

Let us denote the phases of the first and second four-terminal network, respectively, by:

$$\varphi_1 = 2 \text{arc tg} \frac{S}{x_1 - \frac{1}{x_1}} \text{ и } \varphi_2 = 2 \text{arc tg} \frac{S}{x_2 - \frac{1}{x_2}},$$

(4'd)

where

$$x_1 = \frac{F}{F_{01}} \; ; \; x_2 = \frac{F}{F_{02}},$$

(4'e)

F_{01} and F_{02} are the corresponding parametric frequencies.

For $F = F_{av}$ we have:

$$\bar{\varphi}_1 = -\left(\pi - \frac{\theta}{2}\right) = 2 \text{arc tg} \frac{S}{\frac{F_{cp}}{F_{01}} - \frac{F_{01}}{F_{cp}}}$$

(4'f)

and

$$\varphi_2 = -\left(\pi + \frac{\theta}{2}\right) = 2 \text{arc tg} \frac{S}{\frac{F_{cp}}{F_{02}} - \frac{F_{02}}{F_{cp}}}.$$

(4'g)

Solving these equations for F_{01} and F_{02}, we get

$$\left.\begin{aligned} F_{01} &= F_{cp}\left(\frac{1}{2} \frac{S}{\text{ctg} \frac{\theta}{4}} + \sqrt{\frac{1}{4} \frac{S^2}{\text{ctg}^2 \frac{\theta}{4}} + 1}\right) \\ F_{02} &= F_{cp}\left(-\frac{1}{2} \frac{S}{\text{ctg} \frac{\theta}{4}} + \sqrt{\frac{1}{4} \frac{S^2}{\text{ctg}^2 \frac{\theta}{4}} + 1}\right) \end{aligned}\right\}$$

(5)

It is easy to verify that $F_{o1} > F_{av}$ and $F_{o2} < F_{av}$. In addition, it follows from eq. (5) that

$$F_{01} \cdot F_{02} = F_{cp}^2. \tag{6}$$

The quantitative determination of the frequencies F_{o1} and F_{o2} involves the parameter S. The value of the latter must be chosen such that the deviation of the phase difference $\varphi_1 - \varphi_2$ from Θ be a minimum within the frequency band.

To determine the maximum phase deviation, it is necessary first of all to determine the frequencies F_1 and F_2, corresponding to the maximum deviation of the phase difference from the prescribed value of Θ. These frequencies can be obtained from the condition:

$$\frac{d}{dF}\left[2\,\mathrm{arc\,tg}\,\frac{S}{x_1 - \dfrac{1}{x_1}} - 2\,\mathrm{arc\,tg}\,\frac{S}{x_2 - \dfrac{1}{x_2}}\right] = 0. \tag{7}$$

After transformation, eq. (7) assumes the form:

$$(F^2 - F_{cp}^2)(F^4 + \beta\, F_{cp}^2 F^2 + F_{cp}^4) = 0, \tag{7'}$$

where

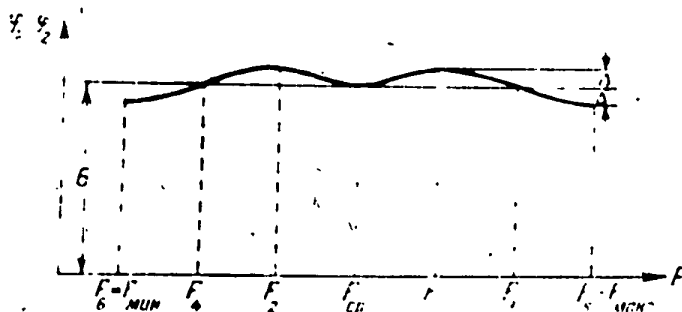$$\beta = 6 - S^2\left(1 - \mathrm{tg}^2\,\frac{\Theta}{4}\right). \tag{7'a}$$

FIG. 3

Eq. (7') breaks up into two equations:

$$F^2 - F_{cp}^2 = 0 \qquad\qquad (7'')$$

and

$$F^4 + 3 F_{cp}^2 F^2 + F_{cp}^4 = 0. \qquad\qquad (7''')$$

The root $F = \overline{F_{av}}$ of eq. (7'') corresponds to the minimum of the function $\varphi_1 - \varphi_2$ (fig. 3), while the roots F_1 and F_2 of eq. (7''') correspond to the maximum of function $\varphi_1 - \varphi_2$:

$$\left.\begin{aligned} F_1 &= F_{cp} \sqrt{-\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} - 1}} \\ F_2 &= F_{cp} \sqrt{-\frac{\beta}{2} - \sqrt{\frac{\beta^2}{4} - 1}} \end{aligned}\right\} . \qquad\qquad (8)$$

The frequencies F_1 and F_2 satisfy the condition $F_1 F_2 = F_{av}^2$. The roots F_1 and F_2 of eq. (7'''), determined from eq. (8), are real when

$$\beta < 0 \qquad\qquad (9)$$

and

$$\beta^2 > 4. \qquad\qquad (9')$$

Thus, in view of inequalities (9) and (9'), the following must be satisfied:

$$\beta = 6 - S^2 \left(1 - \operatorname{tg}^2 \frac{\theta}{4}\right) < 0, \qquad\qquad (9'a)$$
$$\beta > -2$$

or

$$S^2 > \frac{8}{1 - tg^2 \frac{\theta}{1}}.$$

(9'b)

In the general case, the eq. (7''') has real roots when

$$S > \frac{\sqrt{8}}{\sqrt{1 - tg^2 \frac{\theta}{1}}}.$$

(9'')

In the limit, consequently, $S > .83$. The case when $\rho^2 = 4$ and $\rho = -2$ correspond to the roots of (7'') and (7'') coinciding, i.e., $F_1 = F_2 = F_{av}$. Here the phase difference $\varphi_1 - \varphi_2 = \theta$ occurs only when $F = F_{av}$. When $F \neq F_{av}$, the difference $\varphi_1 - \varphi_2 < \theta$.

To obtain the phase characteristic in the form of a double-peak curve (Fig. 3) it is essential that inequalities (9) and (9') be satisfied. Knowing the extremal frequencies F_1 and F_2 it is possible to determine the maximum phase deviation. For this purpose it is convenient to derive an equation for $\left(\cot \frac{\varphi_1 - \varphi_2}{2}\right)_{max}$, obtained after certain transformations as:

$$\left(tg \left(\frac{\varphi_1 - \varphi_2}{2}\right)_{макс}\right) = ctg \left(\frac{\theta}{2} + \frac{\Delta}{2}\right) = \frac{2}{S^2} \sqrt{S^2 \left(1 - tg^2 \frac{\theta}{4}\right) - 4} \; ctg \frac{\theta}{4}.$$

(10)

Eq. (10) establishes the dependence phase deviation Δ on the parameter S when θ is prescribed. Of interest in plotting the phase characteristic are also the frequencies F_2 and F_4, at which the difference $\varphi_1 - \varphi_2$ again becomes equal to θ.

These frequencies can be determined starting with the following considerations:

$$tg \left(\frac{\varphi_1 - \varphi_2}{2}\right)_{F - F_{ах}} = tg \frac{\theta}{2}.$$

(11)

Several values of $F_4/F_{av} = f(S)$ are given in Table 1.

S	$\theta = 30°$ F_4/F_{cp}	$\theta = 60°$ F_4/F_{cp}	$\theta = 90°$ F_4/F_{cp}
3	0,37	0,455	0,55
3,5	0,27	0,312	0,365
4	0,18	0,2	0,23
4,5	0,13	0,142	0,162

Table 1

Inasmuch as the phase characteristic is symmetric about F_{av}, the value of F_3 can be determined from the equation

$$F_3 \cdot F_4 = F_{cp}^2 . \qquad (12)$$

To determine the frequency range of the phase characteristic it is necessary to determine frequencies F_5 and F_6 (fig. 3) at which $\varphi_1 - \varphi_2 = \theta - \Delta$. This can be done with the aid of the equation

$$\text{tg} \left(\frac{\varphi_1 - \varphi_2}{2} \right)_{F=F_{5,6}} = \text{tg} \left(\frac{\theta}{2} - \frac{\Delta}{2} \right) . \qquad (13)$$

For the frequency $F_6 \ll F_{av}$, eq. (13) can be solved approximately. We thus obtain:

$$F_6 \approx \frac{1}{2} \frac{\text{ctg} \left(\frac{\theta}{2} - \frac{\Delta}{2} \right)}{\text{ctg} \frac{\theta}{4}} \times$$

$$\times \frac{S^2}{a} F_{cp} - F_{cp} \sqrt{\frac{1}{4} \frac{\text{ctg}^2 \left(\frac{\theta}{2} - \frac{\Delta}{2} \right)}{\text{ctg}^2 \frac{\theta}{4}} \frac{S^4}{a^2} - \frac{1}{a}}, \qquad (14)$$

(eq. (14) and (16) are real for approximately $S > 3.5$)
where

$$\alpha = S^2 - \frac{S^2}{ctg^2 \frac{\theta}{4}} - 2. \tag{14a}$$

Further, using equation $F_5 F_6 = F_{av}^2$, we get

$$F_5 = \frac{F_{cp}^2}{F_6}. \tag{15}$$

As follows from fig. 3, frequencies F_5 and F_6 are the end points of the frequency range, and therefore $F_5/F_6 = F_{max}/F_{min}$. Eq. (14) and (15) lead to an equation for the above relationship:

$$\frac{F_5}{F_6} = \frac{F_{\text{макс}}}{F_{\text{мин}}} \approx$$

$$\approx \frac{1}{\left[\frac{1}{2} \frac{ctg\left(\frac{\theta}{2} - \frac{\Delta}{2}\right)}{ctg\frac{\theta}{4}} \frac{S^2}{\alpha} - \sqrt{\frac{1}{4} \frac{ctg^2\left(\frac{\theta}{2} - \frac{\Delta}{2}\right)}{ctg^2\frac{\theta}{4}} \cdot \frac{S^4}{\alpha^2} - \frac{1}{\alpha}}\right]^2} \cdot \tag{16}$$

The values of the ratio $F_{max}/F_{min} = f(\Delta)$ are shown in Table 2 for different values of θ.

$\Delta°$	$\theta \cdot 30°$ $F_{\text{макс}}/F_{\text{мин}}$	$\theta \cdot 60°$ $F_{\text{макс}}/F_{\text{мин}}$	$\theta = 90°$ $F_{\text{макс}}/F_{\text{мин}}$
0,5	8,5	6	5,8
1	1	8,5	8,2
2	21	13	12,2
3	31	18,5	17,6
4	41	25,7	23,7

Table 2.

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The analysis given and the equations derived permit selecting, with sufficient accuracy, the basic parameters of the circuit when θ, Δ, or F_{max}/F_{min} are assigned.

The selection of the basic parameter S(10) permits determining $a = \dfrac{1}{S + 2}$ and consequently also the values of C_2, R_2, R_3, etc.

Of greatest interest in single-sideband telephony is the "mean" phase characteristic (fig. 4), where the maximum deviation of the phase difference reaches $\pm$ $\Delta/2$ within the frequency band. Thus, in the given case, the accuracy in maintaining the phase shift is doubled. For the sake of briefness, we shall from now on designate the characteristics of Figs. 3 and 4 by I and II respectively.
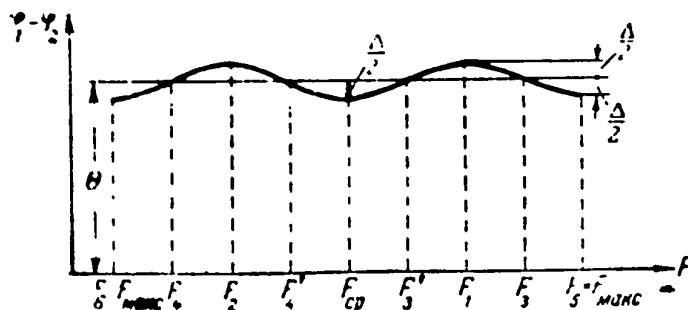

Fig. 4

Analogously with the preceding case, we have for $F = F_{av}$

$$\varphi_1 = -\left[\pi - \left(\frac{\theta}{2} - \frac{\Delta}{4}\right)\right]; \quad \varphi_2 = -\left[\pi + \left(\frac{\theta}{2} - \frac{\Delta}{4}\right)\right] \text{ и } \varphi_1 - \varphi_2 = \theta - \frac{\Delta}{2}. \quad (16a)$$

Hence the expressions for the natural parametric frequencies assume

the form:

$$F_{01} = F_{cp} \left[\frac{1}{2} \frac{S}{ctg\left(\frac{\theta}{4} - \frac{\Delta}{8}\right)} + \sqrt{\frac{S^2}{4 ctg^2\left(\frac{\theta}{4} - \frac{\Delta}{8}\right)} + 1} \right]$$

$$F_{02} = F_{cp} \left[-\frac{1}{2} \frac{S}{ctg\left(\frac{\theta}{4} - \frac{\Delta}{8}\right)} + \sqrt{\frac{S^2}{4 ctg^2\left(\frac{\theta}{4} - \frac{\Delta}{8}\right)} + 1} \right] \qquad (17)$$

The extremal frequencies F_1 and F_2 are determined from eq. (8). However,

we have here $\beta = 6 - S^2\left[1 - \tan^2(\theta/4 - \Delta/8)\right]$.

The maximum phase deviation can be computed with the aid of the equation:

$$ctg\left(\frac{\varphi_1 - \varphi_2}{2}\right)_{\text{макс}} =$$

$$= ctg\left(\frac{\theta}{2} + \frac{\Delta}{4}\right) - \frac{2}{S^2}\sqrt{S^2\left[1 - tg^2\left(\frac{\theta}{4} - \frac{\Delta}{8}\right)\right] - 4} \ ctg\left(\frac{\theta}{4} - \frac{\Delta}{8}\right). \quad (18)$$

Fig. 5 shows the curves of $\Delta/2$ = $f(S)$ for three values of θ. These

curves were computed in accordance with eq. (18).

To plot phase characteristic II, the values of the frequencies F_3, F'_3,

F_4 and F'_4 (fig. 4), at which $\varphi_1 - \varphi_2 = \theta$, may be necessary.
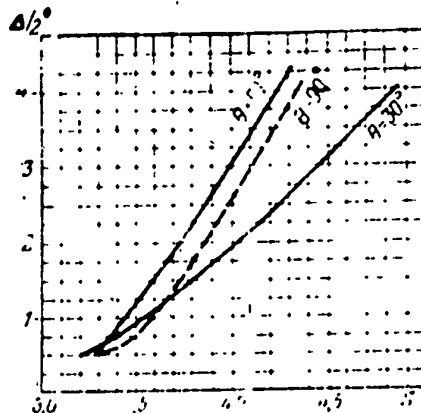

FIG. 5

The equation applicable to these frequencies is (11), which can be approximated by:

$$F \simeq \frac{1}{2} \cdot \frac{\operatorname{ctg}\frac{\theta}{2} \cdot S^2}{\operatorname{ctg}\left(\frac{\theta}{4} - \frac{\Delta}{4}\right) \cdot \alpha_1} \cdot F_{cp} - F_{cp} \sqrt{\frac{1}{4} \cdot \frac{\operatorname{ctg}^2\frac{\theta}{2}}{\operatorname{ctg}\left(\frac{\theta}{4} - \frac{\Delta}{8}\right)} \cdot \frac{S^4}{\alpha_1^2} - \frac{1}{\alpha_1}} \cdot \tag{18a}$$

and

$$F_4' = \frac{F_{cp}^2}{F_4 \cdot \alpha_1}, \tag{18b}$$

where

$$\alpha_1 = S^2 - \frac{S^2}{\operatorname{ctg}^2\left(\frac{\theta}{4} - \frac{\Delta}{8}\right)} - 2. \tag{18c}$$

A more accurate dependence of F_4/F_{av} and $F'4/F_{av}$ on S can be obtained with the aid of the curves of Fig. 6.

Correspondingly, we obtain from the symmetry condition:

$$F_3 = \frac{F_{cp}^2}{F_4} \quad \text{и} \quad F_3' = \frac{F_{cp}^2}{F_1'}. \tag{18d}$$

The equation that permits the end points F_5 and F_6 of the frequency range to be determined can be written as:

$$\operatorname{tg}\left(\frac{\varphi_1 - \varphi_2}{2}\right) F - F_{5,6} = \operatorname{tg}\left(\frac{\theta}{2} - \frac{\lambda}{4}\right). \tag{19}$$

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An approximate solution to eq. (19), taking into account the fact that

$F_5 = F^2_{av}/F6$, yields the relationship

$$\frac{F_4}{F_{\text{a}}} = \frac{F_{\text{макс}}}{F_{\text{мин}}} \approx$$

$$\approx \left[\frac{1}{2} \operatorname{ctg}\left(\frac{\text{H}}{2} - \frac{\Lambda}{4}\right) \cdot S^2 - \sqrt{\frac{1}{4} - \frac{\operatorname{ctg}^2\left(\frac{\text{H}}{2} - \frac{\Lambda}{4}\right)}{\operatorname{ctg}^2\left(\frac{\text{H}}{4} - \frac{\Lambda}{4}\right)} \cdot \frac{S^4}{a_1^2} - \frac{1}{a_1}} \right]^? \cdot$$

(20)

The curves for $\Delta/2 = f(S)$ of Fig. 5 permit us to solve the most frequently encountered problem -- determination of the parameter S when Θ and $\Delta/2$ are assigned. However, one must not exclude the case when F_{max}/F_{min} and Θ are given and it is necessary to determine the maximum possible phase deviation $\Delta/2$, together with the parameter S required for this. Generalized curves for $F_{max}/F_{min} = f(\Delta/2)*$ at various values of Θ are given in Fig. 7. The curves of Figs. 7 and 5 make it possible to determine the parameters of the four-terminal network.
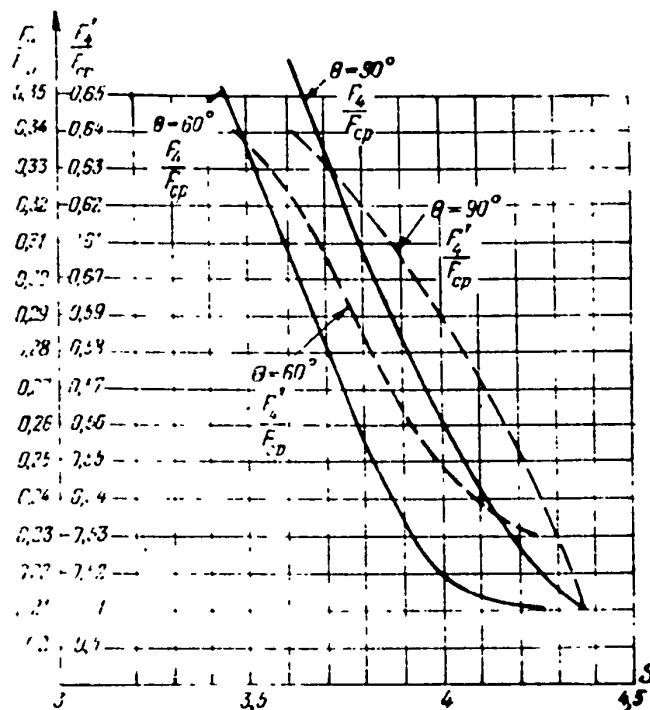

Figure 6

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*The "2" is lacking in original text - Editor.

The computation procedure was already indicated above for the type-I characteristic.

Knowing F_{01}, F_{02}, and a, and prescribing the values of R_1 or C_1, R'_1 or C'_1, it is possible to compute all the necessary elements of the phase-shifting four-terminal networks.

All the above derivations apply when the load resistances R in the anode and cathode circuits are relatively small. Thus, when $R \ll R_1$, the resultant error is negligible. The parameters of the four-terminal network and the value of R must therefore be chosen such that $R/R_1 = 1/50$ to $1/100$; it is therefore clear that it is desirable to choose tubes with low internal resistances and with high plate currents.
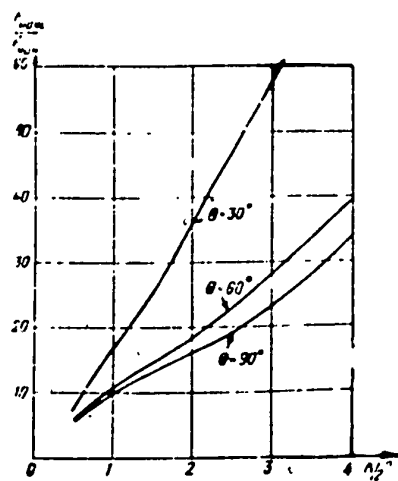

Fig. 7

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3. EXPANDING THE BANDWIDTH OF THE PHASE SHIFTING CIRCUIT.

CASCADE CONNECTION

It follows from the above analysis that if $\theta = 60°$ it is possible to construct a phase-shifting circuit having an accuracy $\Delta/2 = 1°$ in a frequency range $F_{max}/F_{min} = 11$. Correspondingly, for

$$\frac{\Delta}{2} = 2°, \quad \frac{F_{макс}}{F_{мин}} = 18. \tag{20a}$$

However, in broadcasting it is necessary to expand the bandwidth considerably and it may be required to have F_{max}/F_{min} reach 60 to 100.

An effective solution to this problem is cascading the phase shifters in phase opposition. They are so connected that the overall phase characteristic becomes the difference of the phase characteristics of the two phase-shifting circuits. A block diagram of such a cross connection is shown in Fig. 8. In this case the overall phase characteristic equals:

$$\psi = \varphi_1 - \varphi_2 + \varphi' - \varphi_1' . \tag{21}$$
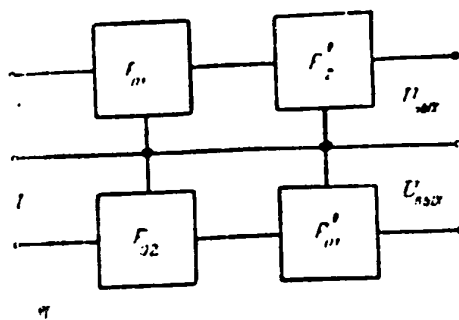

Fig. 8

It is evident that the overall characteristic (type II) cannot assume anywhere over its range values which are beyond $\theta \pm \frac{\Delta}{2}$, i.e., the following condition must be satisfied:

$$\theta - \frac{\Delta}{2} < \psi < \theta + \frac{\Delta}{2}. \tag{22}$$

The most convenient is cascading of phase shifters having symmetric characteristics. It is possible to use for this purpose active phase-shifting networks, which produce symmetric characteristics at various phase-shift angles. Investigations show that phase shifting networks having phase differences θ and $\pi - \theta$ have symmetric characteristics within definite prescribed limits. Such phase-shifting circuit satisfy the condition:

$$S_1^2 \left[1 - tg^2 \left(\frac{\theta}{4} - \frac{\Delta}{8} \right) \right] = S_2^2 \left[1 - tg^2 \left(\frac{\pi - \theta}{4} - \frac{\Delta}{8} \right) \right]. \tag{23}$$

Here S_1 pertains to the phase shifter with a phase-shift angle θ, while S_2 pertains to the one with a phase-shift angle $\pi - \theta$.

Thus it is possible to obtain a phase shift of 120° using opposite-phase excitation with the aid of two phase shifters: one having $\theta = 120^\circ$ and the other with $\pi - \theta = 60^\circ$. The output here is a characteristic with a phase shift of 120° over a wide frequency band (say 100 to 6000 cycles), since each phase-shifting network is designed for this bandwidth.

Eq. (23) together with the above curves and relationships permit design of phase shifting networks for phase shifts equal to $\pi - \theta$. The base points of the characteristics are symmetric in this case.

4. EXAMPLE OF DESIGN OF PHASE SHIFTING CIRCUIT FOR $\theta = 60°$ AND $\frac{\Delta}{2} = 2°$.

We choose a characteristic of type II. From the curves of Fig. 5 and 7 we

obtain $S = 3.74$ and $F_{max}/F_{min} = 18$. Let the minimum frequency of the range

we are interested in be $F_{min} = 200$ cycles; then $F_{max} = 200 \times 18 = 3600$ cycles.

We obtain $F_{av} = \sqrt{F_{min} F_{max}} = \sqrt{200 \times 3600} = 860$ cycles.

Using eq. (17) we find: $F_{01} = 1365$ cycles, $F_{02} = 528$ cycles.

To plot the phase characteristic through the principal points, it is

necessary to compute the values of the frequencies F_1, F_2, F_3, F_4, F'_3, and F'_4

(fig. 4). F_1 and F_2 are determined from eq. (8). Here $\beta = 6 - S^2(1 - \tan^2(\theta/4 - \Delta/8)) = -7.05$. Hence $F_1 = 2270$ cycles, $F_2 = 317$ cycles.

With the aid of the curves of fig. 6 we get: $F_4 = F_{av} \times 0.27 = 232$ cycles;

$F'_4 = F_{av} \times 0.6 = 515$ cycles; $F_3 = F^2_{av} / F_4 = 3050$ cycles; $F'_3 = F^2_{av} / F'_4 = 1380$

cycles.

On the basis of the above, we choose $R_a = R_1/100$ and a 6P3S tube

(permissible plate dissipation 20 watts).

The amplification factor of the tube and the anode and cathode loads

are: $K = \dfrac{\mu}{\mu + 2 + R_4 / R} = 0.77$; $\mu = 60$, $R_k = R_a = 500$ ohm, $R_i = 8000$ ohm.

For class-A operation, $U_{in} \leqslant 30$ volts. Thus, the voltages across the

anode and cathode resistors are $U_R = U_{in} \cdot K = 23.7$ volts. We then have at the

output of the four-terminal network $e_o = U_{R_k} (1-4a) = 7.1$ v., where $a = \dfrac{1}{2+S} =$

0.174.

The elements of the four-terminal networks are computed from the following

relationships: $R_2 = R_1/a$; $C_2 = C_1 \cdot a$; $R_3' = \dfrac{1 - 4a}{4a} R_2$; $C_3 = \dfrac{4a}{1 - 4a} C_2$,

where $R_1 = 100 \cdot R_a = 50,000$ ohm.

5. EXPERIMENTAL SECTION

An experimental study was made of two wide -band phase-shifting networks

having phase shifts of $60°$ and $120°$ and connected with equal-phase inputs. The

range was 100-6000 cycles and $\Delta/2 = \pm 3°$. Both phase-shifting networks

employed 6Zh8 tubes. $R_k = R_a = 800$ ohms, and $R_1/R_a \approx 50$. To reduce the

effect of parasitic capacitors, the capacitors used in the various circuits

were made relatively large.

To provide closer adjustment of the values of the resistances and

capacitances, adjusting elements were added to the phase-shifting-network

circuit. All resistances were measured to within four significant figures.

High-quality and high-stability "styroflex" capacitors were used in the

circuit, having an accuracy of 1-0.5%.

Fig. 9 - Equal-Phase Connection
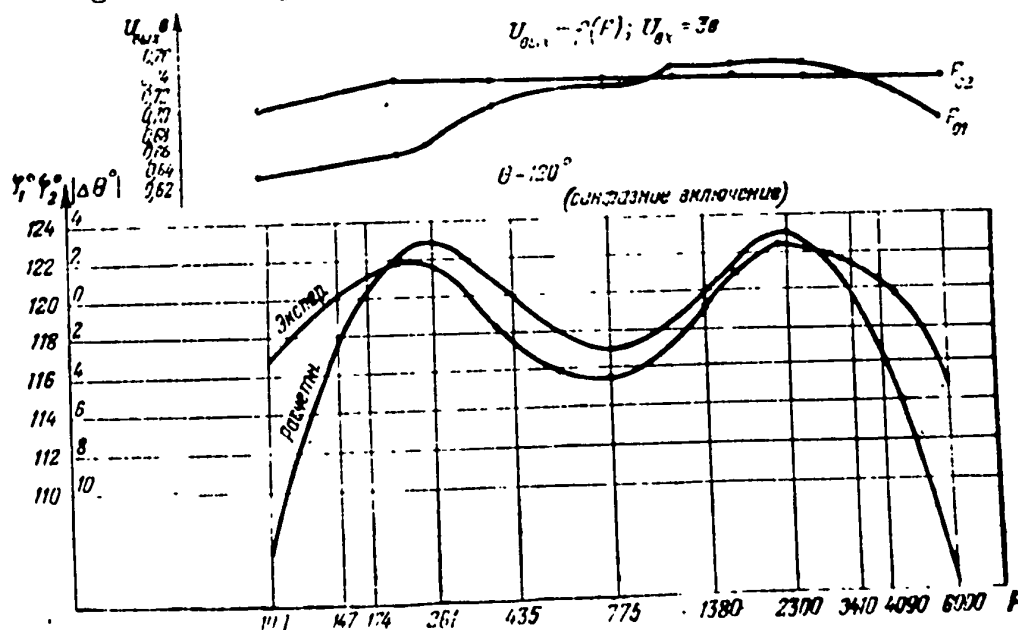
The elements of the phase-shifting networks must be very accurately chosen, for on them depends the constancy with which the phase can be maintained. Thus, for example, for the relatively low-frequency region (x $\ll$ 1):

$$\varphi \approx \operatorname{arctg} \frac{S'}{x - \frac{1}{x}} + \operatorname{arctg} \frac{S''}{x - \frac{1}{x}}, \quad \text{где } S' = S + \delta_{C_1} + \delta_{R_1};$$

$$S'' = S + 2\delta_{C_1} + \delta_{R_2} + 2\delta_{R_1}; \quad \delta_{C_1} = \frac{\Delta C_1}{C_1}, \quad \delta_{R_2} = \frac{\Delta R_2}{R_2}$$

(23a)

etc.

To measure the phase shifts, a special phase meter was constructed, a dual-channel device to whose input the sinusoidal phase-shifted voltages are applied. In each channel the voltage is converted into triangular pulses. The pulses are further shifted in phase and summed across a common load. The resultant d-c component turns out to depend on the phase shift. To increase the sensitivity of the instrument a bucking circuit was used, in which the instrument needle reads zero for $\Theta = 120^\circ$ or 60°. Further needle deflection corresponded to the deviation of the phase characteristic. The phase meter was specially graduated for various frequencies.

For higher accuracy, each measurement was made three times. Before the measurements, the constancy of the potentials across resistors R was checked, and amplitude characteristics were plotted.

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After certain corrections were made to the anode and cathode circuits, the resultant phase characteristics were quite satisfactory and deviated little from the computed values.
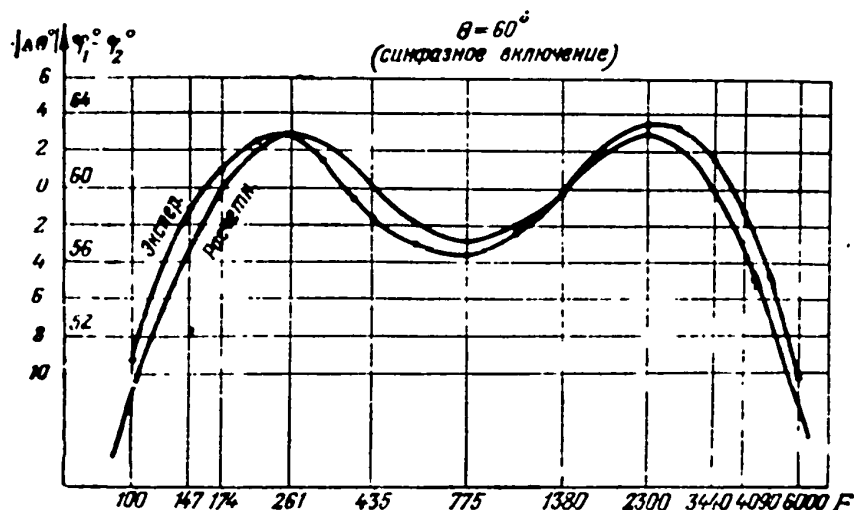
Equal-Phase Connection

Fig. 10

Figs. 9 and 10 contain the phase characteristics for $\theta = 120°$ and $\theta = 60°$ respectively. Fig. 9 contains also the amplitude characteristics $U_{out} = f(F)$.

In conclusion, I thank Professor B. P. Terent'yev for reviewing the manuscript and for valuable comments, and also A. A. Shenovin and V. D. Veynshteyn for help in performing the experiment and planning the work.

Article received by editor 5 May 1952.

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TWO-WIRE THROUGH CONNECTIONS OF HIGH-FREQUENCY TELEPHONE

CHANNELS

N. A. Bayev and

A. P. Resvyakov,

active members of the Society

The article considers the stability conditions of high-frequency

telephone channels both for a single relaying section, as well as

for several sections interconnected by a two-wire through line. It

is shown that a two-wire through connection can be practically equivalent

to a four-wire one, provided certain definite conditions are observed.

In conclusion, the requirements to be met by the low-frequency portion

of the apparatus are given.

1. STABILITY OF HIGH-FREQUENCY TELEPHONE CHANNELS OF A SINGLE

RELAYING SECTION

To determine the stability of a high-frequency telephone channel, it is

possible to use the block diagram of the channel, which is based on an aerial,

cable, or combined (radio and wire) lines (fig. 1).

4-wire through connection

FIG. 1
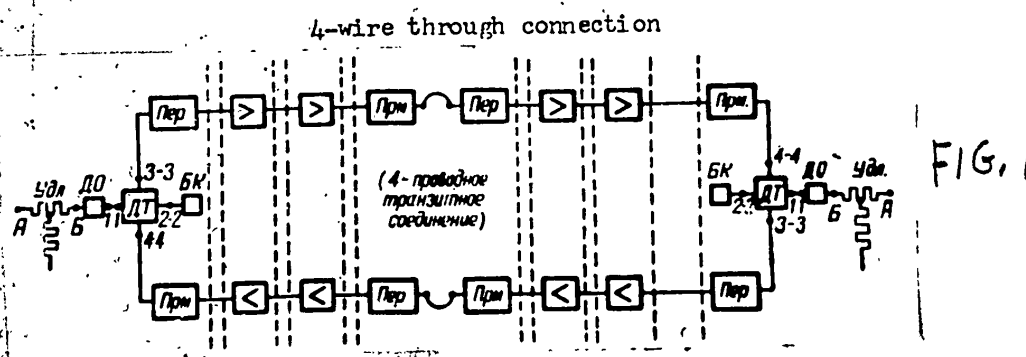
The block diagram above corresponds to the circuit of a single or

several relaying sections interconnected with four wires. In the arguments

that will follow, it will be assumed that the number of intermediate
unidirectional amplifiers connected in series will not affect the stability
of the connection. This means that the stability of the channel in the case
of a single relay section is independent of the distance, and that the stability
of the channel in the case of several relay sections interconnected by four
wires is equal to the stability of the channel of a single relay section.

The extent of the stability, determined on the basis of the conditions
given above, depends in the case of a single relay section on the residual
attenuation, the external load, and the diagram of the low-frequency section
of the apparatus.

The extent of stability is computed, as is known, from the relationship:

$$\sigma = S_{кр} - S_{раб},\qquad\qquad (1)$$

where -- stability, S_{cr} -- critical amplification, S_{op} -- operating amplifica-
tion of the high-frequency telephone channel for one relay section.

When determining the operating amplification S_{op} it is necessary to
take into account the external load and the diagram of the low-frequency
portion of the apparatus.

When the stability is determined experimentally, its value is defined
by the relationship

$$\sigma_{эксп} = -\frac{b_{r1}+b_{r2}}{2} - \frac{b'_{r1}+b'_{r2}}{2},\qquad\qquad (2)$$

where b_{exp} -- stability as determined experimentally; b_{r1} and b_{r2} -- residual
damping of the high-frequency channel, measured in the forward and reverse
directions, with the nominal load replacing the operating load, and at the

same time with the feedback not being taken into consideration; b'_{r1} and b'_{r2} —

residual attenuation of the high frequency channel, corresponding to oscillation;

they are established in the presence of feedback currents, but are measured

in the absence of the latter.

It is evident that the stability values determined theoretically and

experimentally should be the same. However, when the stability is theoretically

determined the effect of feedback currents is usually not taken into account

and all that is determined is the nominal stability; which agrees with the

experimentally-determined stability only under certain conditions, namely:

$$\sigma_\mu = S_{Kp} - S_\mu. \tag{3}$$

where b_n is the nominal stability, i.e., the stability determined by

computation without taking feedback into account; S_{cr} — critical amplifica-

tion, defined as half the sum of the balanced damping; S_n — operating amplifi-

cation of the channel disregarding the effect of the feedback currents (under

nominal load).

On the other hand, if the feedback currents are taken into account,

it is necessary to use the actual amplification S_a instead of S_n; the value

of the stability then becomes

$$\sigma_\phi = S_{\mu\rho} - S_\phi. \tag{4}$$

where b_a — actual stability, i.e., the stability as determined by computa-

tion with the effect of the feedback currents taken into account.

S_a, in turn, is obtained from the expression

$$S_\phi = [S_\mu + \Delta S] \div [S_\mu - \delta S], \tag{5}$$

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where ΔS is the positive increment of the nominal amplification due to the effect of the feedback currents, while δS is the negative increment in the same quantity.

To be able to change over from the nominal stability to the actual one, it is necessary to evaluate the effect of the feedback currents. The limiting changes ΔS or $\delta S^{(1)}$ in the nominal amplification of the channel, due to the presence of feedback currents, are shown in fig. 2. Plotted along the abscissa of fig. 2 is the value of the nominal stability σ_n of the quantity $x = b_n - 2\sigma_n$, i.e., the extent of attenuation in the loop.

The value of the actual stability σ_a, or more accurately, of its upper limit, is easily determined from eq. (5), (4), and (3):

$$\Delta S = S_\phi - S_z = S_{xp} - S_x - \sigma_\phi = \sigma_x - \sigma_\phi, \qquad (6)$$

or

$$\sigma_\phi = \sigma_x' - \Delta S. \qquad (6a).$$

The nominal stability of the channel of a single relay section, both open-circuited or short-circuited, is equal to the residual damping and does not depend on the attenuation of the lengthening/lengthening coil.

To prove the correctness of this assumption, it is enough to determine the stability in accordance with the circuit shown in fig. 1. If the residual damping b_r will be connected between points A-A, the damping of the set of lengtheners to either side will be $b_L = \frac{b_r}{2} + b_{st}$, and the characteristic

impedance will equal the input impedance of the balanced equipment, then, the amplification effective at points B-B should be $S_n = 2b_L - b_r = 2b_{st}$ (b_{st} is the attenuation of the supplementary station lengthening coil).

With this as a start, the value of the nominal open-circuited (or short-circuited) nominal stability is determined as

$$\sigma_n = S_{Kp} - S_n = b_e - S_n = b_r, \quad (7)$$ (7)

where

$$S_n = 2b_y' - b_r = 2b_{cm},$$ (7a)

and the balanced attenuation is

$$b_e = \ln\left[\frac{Z \, cth \, b_y + Z}{Z \, cth \, b_y - Z}\right] = \ln\left[\frac{1 + th \, b_y}{1 - th \, b_y}\right] = 2b_y.$$ (7b).

Since expression (7) is general in form, and is valid for any value of the lengthener attenuation b_L, starting with zero.

The nominal stability σ_n of one relay section can therefore not be increased by introducing additional lengthening coils.

The value of the experimentally-determined stability σ_{exp} can correspond not only to the nominal stability σ_n, but also to the actual stability σ_a. Actually, if it is assumed that in eq. (2) $b_r' = b_r - S_n$, and $b_r = S_n$, then only in that case is $\sigma_{exp} = \sigma_n$, but since

$$b_r' = b_r - S_\phi = b_r - [S_n + \Delta S] = -\Delta S,$$ (8)

then $\sigma_{exp} = \sigma_a$ [see (2), (5), and (6)].

Fig. 2

2. STABILITY OF HIGH-FREQUENCY CHANNEL IN CASE OF TWO-WIRE

THROUGH CONNECTION

To determine the stability of the channel in the case of a two-conductor
through connection, it is possible to employ the concept of nominal stability,
inasmuch as the relationship between the nominal and actual stabilities has been
determined above. Usually, to maintain the same residual attenuation of the
resultant connection, two so-called through lengtheners, each having an
attenuation $b_r/2$, are disconnected at the point of connection of the terminal
apparatus.

However, disconnecting these lengtheners reduces the stability of the
resultant connection. This does not contradict the statement made above that

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it is impossible to increase the stability of the apparatus of a single relay section by introducing lengtheners. Actually, the value of the nominal stability σ_n of the channel (see eq. (1) and (2)) through-connected with a two-conductor line, can be determined as:

$$\sigma_{нд} = \frac{-\ln\left[e^{-b_{сr}} + e^{-b_r}\right] + b_r}{2} = \frac{-\ln\left[\Delta + e^{-b_r}\right] + b_r}{2} \tag{9}$$

in the case of two relay sections, and as

$$\sigma_{нд} = -\ln\left[\Delta + e^{-b_r}\right] \tag{10}$$

for the middle one of three relay sections; here b_{con} is the mismatch attenuation at the connection point between two through-connected relay sections, and Δ is the corresponding reflection coefficient[1].
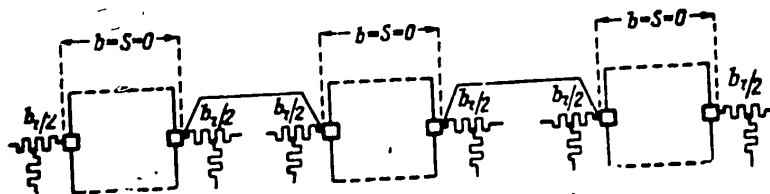

Fig. 3

The quantity that indicates the reduction in stability when the number of relay sections through-connected by two wires is increased, as compared with

the stability of a single relay section, i. e., with the stability of a four-wire through-connection system, can be determined as follows:

$$\Delta\sigma_{N3} = \sigma_{N1} - \sigma_{N2} = \frac{\ln\left[1 + \Delta e^{b}r\right]}{2} \tag{11}$$

for one of two relay sections, or

$$\Delta\sigma_{N3} = \sigma_{N1} - \sigma_{N2} = \ln\left[1 + \Delta e^{b}r\right]. \tag{12}$$

for the middle one of three relay sections.

For the resultant connection to operate stably, it is desirable that expressions (9) and (10) be as large as possible, and expressions (11) and (12) be as small as possible (fig. 4).
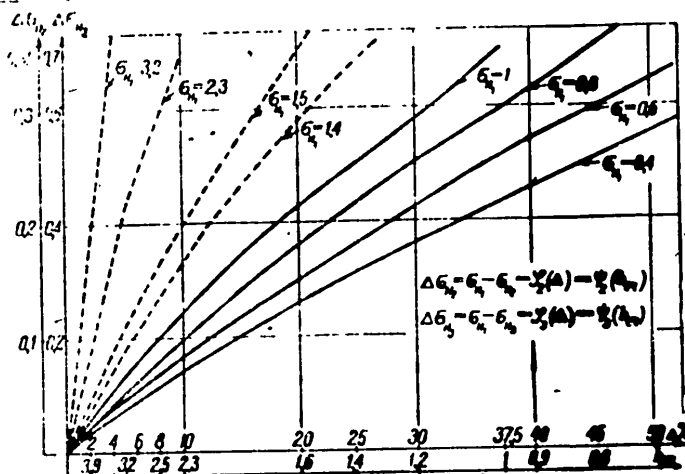

Fig. 4

If the reflection coefficients at the junctions is zero ($\Delta = 0$), then each of e.(9) or (10) will be equal to th value of b_e — to the channel stability for a relay section or to the stability of the channel in the case of four-wire through connection, while e. (11) and (12) will equal zero.

If the reflection coefficients at the junction points will be on the order of $\Delta = 0.02$ to 0.05 or less ($b_{av} = 3.5$ to 3 nepers), then the two-wire through connection will be practically equivalent to the four-wire line as far as stability is concerned.

To attain this goal, it is necessary to standardize the input impedance of the low-frequency portion of the apparatus, reducing its deviation from the nominal value by reducing the reflection coefficients between the differential hybrid systems, or by introducing station lengthening coils to supplement those used in the through connections.

The reduction in stability in two-wire through connections depends not only on the reflection coefficients at the junction points, but also on the value of the selected residual attenuation. This is particularly important in the design, because for given reflection coefficients the decrease in the stability will increase with the amount of installed residual attenuation of the individual connections (fig. 4).

If supplementary station lengthening coils are introduced into the two-wire part of the line, the stability of each relay section will not increase, as discussed above, but the stability of the resultant connection will increase considerably in the case of a two-wire through connection.

If lengthening coils of a definite value are used, the two-wire through connection can be practically equivalent to the four-wire connection.
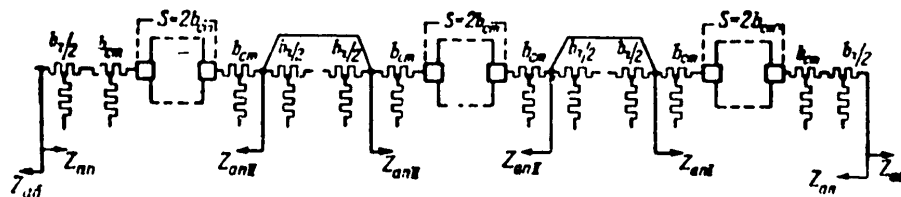
Fig. 5

To determine the stability of the channel in this case (fig. 5), it is possible to write the following relationships:

$$\delta'_{\mu_s} = \frac{b_r + 2b_{cm} - \ln\left[e^{-(b_{cr} + 4b_{cm})} + e^{-(b_r + 2b_{cm} + 4b_{cm} - 4b_{cm})}\right] - 4b_{cm}}{2} =$$

$$= \frac{b_r - \ln\left[\Delta e^{-2b_{cm}} + e^{-b_r}\right]}{2} \tag{13}$$
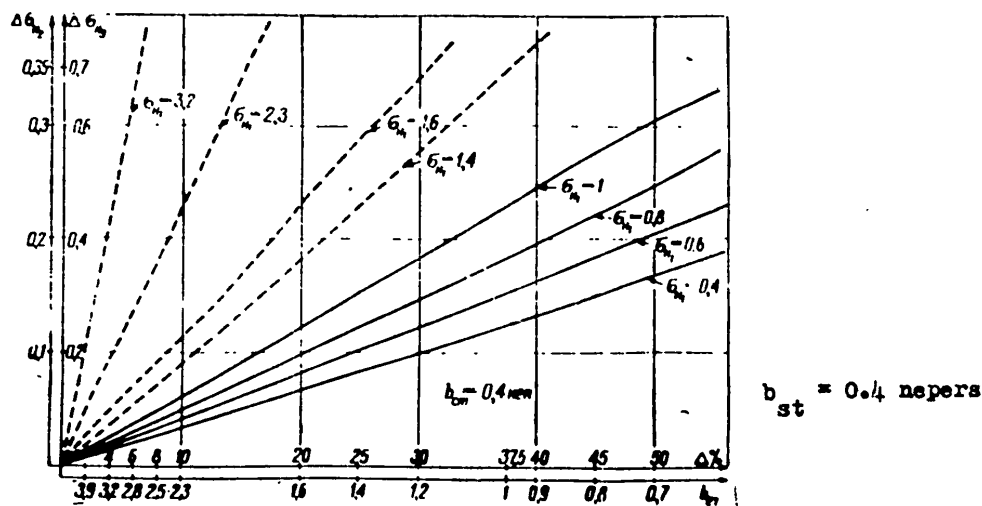

$b_{st} = 0.4$ nepers

Fig. 6

for the channel in the case of two relay sections, or

$$\sigma'_{N_a} = -\ln[\Delta e^{-2b_{cm}} + e^{-b_r}] \qquad (14)$$

for the middle one of three relay sections.

The extent of reduction in channel stability when it is introduced into a two-wire through connection can be determined from the expression

$$\Delta\sigma'_{N_a} = \frac{\ln[1 + \Delta e^{b_r - 2b_{cm}}]}{2} \qquad (15)$$

for one of two relay sections, or

$$\Delta\sigma'_{N_a} = \ln[1 + \Delta e^{b_r - 2b_{cm}}] \qquad (16)$$

for the middle one of three relay sections.

For stable operation of high-frequency communication equipment using two-wire through connections, if reflection (Δ = 0.2) exists at the junctions, it is necessary to employ auxiliary station lengtheners of not less than 0.4 neper (fig. 6) or 1 neper (fig. 7).
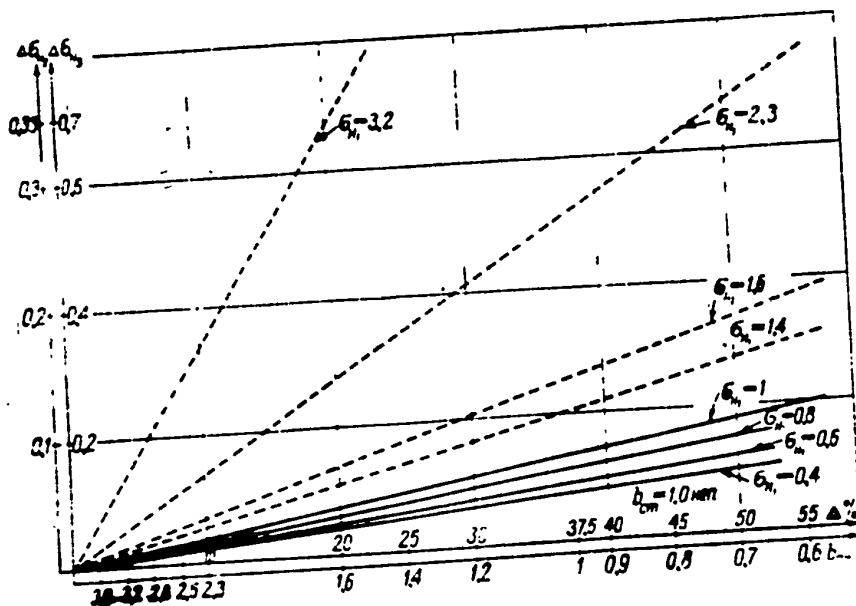
Figure 7

We shall consider below two methods that permit reducing the reflection coefficient of the input devices of the low-frequency portion of the apparatus, so as to insure stable operation when two-wire through connections are used. Reducing the reflection coefficients of the input devices of the apparatus permits increasing the mismatch attenuation between the low-frequency portions of the apparatus, connected by means of two-wire through connections (fig. 5):

$$b_{er} = \ln \frac{1}{\Delta} = \ln \left| \frac{Z_{an}I + Z_{an}II}{Z_{an}I - Z_{an}II} \right|, \qquad (17)$$

or between the low-frequency portion of the apparatus and the subscriber's line:

$$b'_{er} = \ln \frac{1}{\Delta'} = \ln \left| \frac{Z_{an} + Z_{a6}}{Z_{an} - Z_{a6}} \right|. \qquad (18)$$

To determine the amount of mismatch attenuation. it is necessary to know the input impedance of the apparatus (points 1-1) of the differential system, and also to have data concerning the supplementary instrument connected between the indicated points and the external terminals (A-A) of the apparatus (fig. 1).

Let us determine the input impedance of the differential system, disregarding the supplementary equipment.

In this case the input impedance of the differential system can be determined

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from the general expression for the input impedance of a four-terminal network:

$$Z_1 = \frac{A_1 R_N + B}{C R_N + A_2} = \frac{n^2 Z_3 (Z_3 + 4Z_4) + 4Z_4 Z_3}{n^2 Z_3 + 4(Z_4 + Z_3)}. \tag{19}$$

Here A_1, A_2, B and C are the four-terminal-network coefficients, related by the equation $A_1 A_2 - BC = 1$;

R_L -- load resistance of the four-terminal network under consideration;

Z_1 -- input impedance of the four-terminal-network investigated at terminals 1-1;

n -- transformation coefficient of the differential transformer;

Z_2, Z_3, Z_4 -- impedances connected to terminals 2-2, 3-3, and 4-4 of the differential transformer;

Z_0 -- characteristic impedance of differential system.

If the load impedances are properly selected, i.e.,

$$Z_3 = \frac{2Z_0}{n^2}; \quad Z_2 = Z_0 \text{ и } Z_4 = \frac{Z_0}{2}, \tag{20}$$

then Z_1 will equal Z_0 as a result of (19).

If only one of the load impedances, Z_3 or Z_4, differs from the nominal value by m times, it follows from (19) that the input impedance at terminals 1-1 will equal

$$Z_1 = Z_0 \frac{3m+1}{m+3}. \tag{21}$$

If both load impedances Z_3 and Z_4 differ from the nominal by m times (identical shop tolerances), then

$$Z_1 = Z_0 m. \tag{22}$$

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Finally, if load impedances Z_3 and Z_4 differ from their nominal values by m and $1/m$ times respectively, i.e., mZ_3 and $\frac{1}{m}Z_4$, or Z_3/m and mZ_4, then the input impedance is

$$Z_1 = Z_0. \qquad (23)$$

Thus, by satisfying conditions (20) or (23) it is possible to obtain a sharp reduction in the reflection coefficient between the input impedance of the apparatus, and the load replacing the subscriber line, or the input impedance of the apparatus that is being through-connected. Similar results can be obtained by introducing through-line lengthening coils in addition to station lengthening coils (fig. 5).

The effect of the supplementary circuit, which is connected into the low-frequency portion of the apparatus, must be taken into account, by determining the deviation of the input impedance from the prescribed value.

If the supplementary circuit acts (for most of the spectrum) like a four-terminal network with a characteristic impedance equal to the characteristic impedance of the differential circuit and the lengthening coil, then it is necessary to add to the attenuation of the lengthening coil the value of the attenuation of the above-mentioned four-terminal network. The value of the stability remains almost unchanged in this case.

If the auxiliary circuit forms an incomplete terminal network, then the stability may change — decrease or increase, depending on the external circuit and its operating conditions (open-circuit, short-circuit, connected to operating load, etc.). The supplementary circuit will leave the stability practically

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unchanged if the relationship between it and the balanced circuit will correspond to a mismatch attenuation b_m (sup) $\geqslant$ 3.5 nepers.

For stable operation of a high-frequency channel, through-connected in a two-wire circuit, it is necessary to have at the junction point a reflection coefficient of 0.07 to 0.1. To attain this, it is necessary to have a reflection coefficient not exceeding 0.03 to 0.05 between the input impedance of the apparatus (without a through lengthening coil, but with a station lengthening coil) and a 600 ohm load.

In conclusion, it must be noted that if the above values of reflection coefficient are not obtained (either by improving the input impedance of the apparatus or by connecting station lengthening coils), then the two-wire through-connection will be of low quality and must be replaced with a four-wire system.

Article received by editor 31 January 1953.

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CONCERNING A. A. KHARKEVICH'S ARTICLE ON "DETECTION OF WEAK SIGNALS"

Engineer A. K. Rozov has addressed the editor with a question concerning the article by A. A. Kharkevich on "Detection of Weak Signals," published in the periodical Radiotekhnika /Radio Engineering/, No. 5 of 1953.

A. K. Rozov asks: "Which of the two methods for detecting weak signals, discussed in the above article -- correlation or accumulation method -- requires a longer time of transmission of the useful signal? Does the correlation method offer a possibility of reducing the required signal-detection time as compared with the accumulation method when the same input and output signal-to-noise- conditions prevail? If so, how is this reconciled with the principle of "incompressibility" of the signal and with the well-known theorem on the connection between the power spectrum $\omega(f)$ and the correlation function?"

The text of A. A. Kharkevich's answer to this letter is given below.

Direct comparison of the expectation time for the accumulation and correlation methods is impossible because the noise properties are not equally defined in both methods. In the accumulation method noise is described by its mean-squared value, σ . In the correlation method, on the other hand, the situation is additionally described by the correlation function $r_{\xi\xi}(\tau)$ of the noise. The expectation time required to make $R_{\xi\xi}$ and R_{xx} of the comparable order of magnitude can be determined from the equation:

$$\frac{\sigma^2}{a^2} r_{\xi\xi}(\tau) \approx 1$$

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(it is assumed that $r_{xx} \approx 1$). This equation must be solved with respect to τ , and for this it is necessary to know the correlation function. Instead of the correlation function, it is possible to prescribe the noise power spectrum, inasmuch as the two functions are uniquely related by a Fourier transform pair.

Concerning the possibility of gaining and the concept of "incompressibility," the latter must not be considered as simplified. In any signal transformation there exists a certain transformation invariant, but the "volume" of the signal is not necessarily this invariant. Speaking more simply, whenever any "measure" of a signal is reduced (i.e., its duration or the spectrum width or the dynamic range), another measure (or measures) is increased, but the "volume" of the signal may change at the same time. To make a mechanical analogy, it can be said that the situation with the deformation of the signal is analogous to that prevailing in the deformation of an elastic body; for example, in the case of a longitudinal compression we observe a transverse expansion, but this expansion is lower than would take place were this body incompressible, so that the volume of the body changes nevertheless. In the theory of elasticity, the relationship between the transverse expansion and the longitudinal compression is given by the Poisson equation. Similar coefficients can also be introduced to describe the deformation of signals; these coefficients would be different for various types of transformations.

PROFESSOR S. I. KATAYEV'S FIFTIETH BIRTHDAY

In February of this year the Moscow Electrotechnical Communications Institute honored the head of the television department, Doctor of Technical Sciences, Professor Semyon Isidorovich Katayev on his fiftieth birthday and the twentieth anniversary of his scientific and pedagogic work.

S. I. Katayev is one of the leading scientists working in the field of modern means of electronic television and pulse techniques.

S. I. Katayev was born in 1904. In 1929 he was graduated from the Moscow electrotechnical school, presenting as his thesis project his first theoretical investigation, devoted to energizing radio-receiving devices from thermocouples.

Further work by Semyon Isidorovich was devoted to electronic television systems and to pulse techniques. In 1931 he invented a device for television transmission containing all principal properties of the modern iconoscope, and he attained the establishment of the development of this tube in the All-Union electrotechnical institute (VEI).

In 1932 the periodical Tekhnika Svyazi /Communication Engineering_/ published an article by S. I. Katayev, establishing the advantages of electronic television system over mechanical ones. In the same article he emphasized that the transmitting television system must employ the storage effect and cited as an example a simple transmitting cathode-ray tube, proposed by him, with a one-sided mosaic, which permitted practical realization of the principle of charge accumulation.

In another of his suggestions, dating back to 1932, S. I. Katayev pointed out the advantages of employing in cathode-ray tubes the transfer of the image electron from a conducting photo-cathode onto a dielectric. This idea, further developed by P. V. Timofeyev and P. V. Shmakov (1933) has been realized in modern tubes with image transfer.

Under the leadership of Semyon Isidorovich, the first high-vacuum magnetic-focusing receiving tube was also developed in 1932; this tube replaced the gas-focusing tubes employed at that time.

In 1932 the Svyaz'izdat published a monograph by S. I. Katayev, "Cathode-Ray Television Tubes", in which along with the attainments in electronic television, there is an exposition of the essence of the investigations made by the author on the role of secondary electrons in cathode-ray television tubes. The dependence on various factors, observed by him, of the potential of an insulated electrode bombarded with electrons, remains to this day one of the most important basic premises in the design of transmitting and receiving tubes.

From 1937 to date S. I. Kitayev has continuously been in charge of the television faculty of the Moscow Electrotechnic Communications Institute.

In 1940 Svyaz'izdat published a book, Osnovy Televideniya /Fundamentals of Television_7, edited by S. I. Katayev and written by a group of authors. Individual chapters of the book were written by Semyon Isidorovich. This book withstood the test of time -- for 14 years it has been the basic text book for higher communication schools.

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In a monograph published in 1951 by Gosenergoizdat, "Generatory impulsov televisionnoy razvortki" /Pulse Generators for Television Scanning_7, I. S. Katayev gives an original "mosaic" method for constructing simplest relaxation oscillators producing electric pulses of prescribed form, and an analysis of a self-excited sawtooth oscillator.

The problems of pulse technique, to which S. I. Katayev devotes great attention since 1936, have found expression in many pulse generators of arbitrary wave form, invented by him, in several lectures, and also in a doctoral dissertation "Problem of obtaining electric pulses of any form," which he submitted to the Moscow Electrotechnic Communications Institute in 1951.

S. I. Katayev carries on considerable work in the popularization of modern attainments of leading Soviet science among the wide strata of the population. The All-Union Society for Propagation of Political and Scientific Knowledge, of which he is an active member, has published many copies of lectures that he delivered in the lecture hall of the Polytechnical Museum, and organizes systematically lectures on various problems of television engineering.

Along with scientific and pedagogical activities, S. I. Katayev carries on considerable social work; he is a member of the board of the All-Union Scientific-Technical Society of Radio Engineering and Electric Communications imeni A. S. Popov, director of its television section, and also a member of the editorial board of the periodical "Radiotekhnika" /Radio Engineering_7.

S. I. Katayev was awarded the order of the Labor Red Banner, with three medals, and with an honorary diploma by the Uzbek SSR.

Reaching his fiftieth birthday in full bloom of his creative forces, S. I. Katayev devotes all his efforts to the development of the Fatherland's science.

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AT THE ALL-UNION SCIENTIFIC TECHNICAL SOCIETY OF RADIO ENGINEERING AND ELECTRIC COMMUNICATIONS IMENI A. S. POPOV (VNORIE)

The All-Union Soviet of scientific engineering-technical societies has completed the Society's work program for 1954, presented by the Board of the VNORIE. Particular attention was devoted in the program to the question of participation of the Society in cooperating with the development of electric-communications and radio-installation means in rural localities. It has been decided to carry on at various localities joint conferences with the Communications Ministry and with other interested organizations; these conferences would be devoted to problems of development of intra-rayon communication and internal-operation communication in the kolkhozes and the machine-tractor stations. Consideration will also be given to problems in operating kolkhoz radio centers and to radio installations in the farms.

In accordance with resolution by the party of the government, the industry is faced with the problem of expanding the manufacture of marketable products for mass-consumption, and improving its quality. To aid the industry, the plan of the industry provides for carrying out many conferences and meetings for wide evaluation of the quality of the apparatus and of the assembly parts, issued by the industry, and the development of suggestions for their improvement.

Scientific-technical conferences, which will be held by the Society jointly with interested organizations, will deal with evaluation of the qualitative indices of tape recorders for popular use, and to consider

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problems associated with selecting sound-carrier systems; particular
examination will be made of the qualitative indices of long-playing
records and of devices for their playing.

Provision is made for measures contributing to interchange of
scientific-technical practical experience. Serving this purpose should be
the scientific technical conferences, devoted to the celebration of the
Radio Day, which will be held locally by branches of the Society, and also
a series of other measures, among them being: conferences devoted to problems
of automatization and mechanization of manufacture, to problems of development
and manufacture of loudspeakers for radio receivers, to operating experience
with communication equipment, and automatization of electric-transmission
lines, etc. Work will continue on the introduction of advanced labor methods
in related enterprises.

An important task is the raising of the scientific-technical level of
members of the Society. The plan provides for lectures on scientific-technical
topics, organization of seminars and of study circles. In particular, it is
intended to organize in Moscow a lecture series on the statistical theory of
communications, and others.

A considerable section of the plan is devoted to the question of
participation by the members and by the organizations of the Society in the
socialist competition for better results in the adoption of advanced techniques
in manufacture.

The work program of the Society provides for many discussions, in
which the most important scientific-technical literature on radio and
electric-communication problems will be examined on the basis of widely-developed

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criticism and self-criticism; there will also be held readers' conferences, where the contents of the periodicals "Radiotekhnika", "Radio" and others will be reviewed.

The most important measure will be to prepare and hold the second congress of the members of the Society to elect the leaders of the Society organizations. The congress will be held in Moscow in May 1954, directly after completion of the All-Union scientific session devoted to the celebration of Radio Day.

It is planned to hold a scientific-technical conference in the second quarter of this year; this conference will have as its purpose to evaluate the basic problems associated with the improvement of sound quality in radio and television receivers, and to define the scope of further improvement. Invited to participate in the work of this conference are enterprises and scientific-research organizations of the radio industry, of the Ministry of Communications, and of the Ministry of fuel and local industry of the Ukrainian, Belorussian, Russian, Azerbaijan, and Latvian SSR's.

The Society Board has organized, together with the USSR Academy of Sciences Laboratory for study of scientific problems of wire communication, a series of lectures on the theory of relay-contact circuits and their application to the development of automatic-communication circuits. The program of the lecture series is scheduled for 40 hours. The first lectures were already given.

The Society Board held a conference devoted to intercity television networks. The conference discussed problems of television transmission over cables, and also norms for television channels.

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Many lectures were devoted to methods of narrowing-down the frequency spectrum in intercity television, and methods for correcting television signals. Also considered were problems pertaining to rebroadcast of television programs and to long-distance reception of television broadcasts.

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of transitional processes, distortions of signals are investigated in linear radio circuits under varying types of modulation. Methods are described of a calculation suitable for solitary signals, both periodic and non-regular.

The book is intended for students and aspirants in the technical radio specialties, as well as for engineers and scientific workers in the fields of radio engineering and radio physics.

L. I. Bayda and A. A. Semenkovich. "Elektronnyye usiliteli postoyannogo toka" (DC Electronic Amplifiers). Gosenergoizdat, Moscow-Leningrad, 1953, 191p., price 7 rubles 75 kopecks.

The author considers individual types of electronic and ion amplifiers which have received basic application in automatics and in electrometering technology. He describes the features of these amplifiers, gives an analysis of their operating regimes, and explains the action of the circuits. The book is intended for engineering-technical workers who need to be concerned with questions of amplifier use in connection with automatics and construction of devices, and should also prove useful for university students of the pertinent disciplines.

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L. S. Gutkin. "Preobrazovaniye sverkhvysokikh chastot i detektirovaniye" (Transformation of UHF and Detection). (Questions of Theory and Computation). Gosenergoizdat, Moscow-Leningrad, 1953, 415p. price 16 rubles 80 kopecks.

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The author examines the most important problems in the computation of transformers of UHF and detectors, being applied in radio receiving equipment. Main attention is devoted to the following problems: detection of impulse signals, the interaction of a signal and noise during detection, the transformation of frequency into the range of metric, decimetric, centimetric waves etc.

The book is intended for scientific personnel, engineers and students of advanced radio courses.

Ya. L. Al'pert, V. L. Ginzburg and Ye. L. Feynberg. "Rasprostraneniye radiovoln" (Propagation of Radio Waves). Gostekhizdat, Moscow, 1953, 883 p., price 33 rubles 65 kopecks.

In the book are examined the main questions of the theory and experimental investigations of propagation of radio waves over the entire radio range--from micro-radio waves to long waves. The book is designed for students in advanced courses, aspirants, engineers and scientific personnel. It may be used both as a profound study of the question and for investigatory work, as well as for practical computations.

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Correction to the Journal "Radio Engineering", No. 1, 1954:

On page 74 in the 2nd column, 13th line from the top, the following words are missing: "For the Department of the Physico-mathematical Sciences, N. N. And STAT was selected as Academician, while L. M. Brekhovskikh was chosen as Member-correspondent."